SECTION 3-4: DERIVATIVES AS RATES OF CHANGE

1. Suppose $p(t)$ gives the number of bacteria in hundreds after t hours in some lab experiment.

   (a) Interpret $p(10) = 1000$ and $p'(10) = 20$.

After 10 hours, there are 100,000 bacteria in the population and the population is increasing at a rate of 20 bacteria per hour.

   (b) Estimate the number of bacteria when $t = 11$.

$p(11) \approx p(10) + p'(10) = 1000 + 20 = 1020$ hundreds of bacteria.

2. Suppose $s(t)$ gives the position of an object where s is measured in feet and t is measured in seconds.

   (a) Determine the units of $s'(t)$ and $s''(t)$ and interpret them in the context of the problem.

units of $s'(t)$: ft/sec

units of $s''(t)$: ft/sec/sec $= ft/s^2$

What is the difference between speed and velocity?

speed = |velocity|

   (b) Can $s'(t)$ be negative? What would that mean?

$s'(t)$ can be negative.

The object is moving from a higher position to a lower position.

   (c) If $s'(5) = 20$ and $s''(5) = 2$, estimate $s'(6)$. Is the object speeding up or slowing down?

$s'(6) \approx s'(5) + s''(2) = 20 + 2 = 22$ ft/s (speeding up)

   (d) If $s'(5) = 20$ and $s''(5) = -2$, estimate $s'(6)$. Is the object speeding up or slowing down?

$s'(6) \approx s'(5) + s''(5) = 20 - 2 = 18$ ft/s (slowing down)

   (e) If $s'(5) = -20$ and $s''(5) = -2$, estimate $s'(6)$. Is the object speeding up or slowing down?

$s'(6) \approx s'(5) + s''(5) = -20 - 2 = -22$ ft/s

(speeding up)

3. A potato is launched vertically upward from a platform 20 feet off the ground. The distance in feet that the potato travels from the ground after t seconds is given by $s(t) = -16t^2 + 64t + 20$.

(a) Find the initial velocity of the potato.

Initial velocity $= v(0)$. $s'(t) = v(t) = -32t + 64$

$s'(0) = 64$ ft/s

$$\begin{array}{r} 128 \\ -\ 64 \\ \hline 84 \end{array}$$

(b) Find the velocity and the acceleration of the potato when $t = 1$.

$s'(1) = -32(1) + 64 = 32$ ft/s

$s''(t) = -32$. $s''(1) = -32$ ft/s^2

$$\begin{array}{r} 2 \\ 16 \\ 4 \\ \hline 64 \end{array}$$

(c) Is the potato speeding up or slowing down? Why?

Slowing down.

s' and s'' have different signs.

(d) What is the velocity of the potato when it reaches its maximum height and why?

$v(t) = 0$ at maximum height. This is the moment when change in position ($v(t)$) switches from + to −.

(e) What is the maximum height of the potato?

$s' = -32t + 64 = 0$

when $t = 2$. ← WHEN

max height when $t = 2$:

$s(2) = -16(2)^2 + 64(2) + 20 = 64$ ft

(f) Assume the potato lands on the ground (not the platform). How long is the potato in the air?

Find t when $s = 0$.

So $0 = -16t^2 + 64t + 20$ → Use quad formula

$$t = \frac{-64 \pm \sqrt{64^2 - 4(-16)(20)}}{2(-16)} = \frac{1}{2}(4 + \sqrt{21}) \approx 4.2913$$ ft/sec

(g) What is the velocity of the potato when it hits the ground?

$v(4.2913) = -32(4.2913) + 64 = -73.32$ ft/sec

(h) You should have observed in part (b) that the acceleration is constant. What does this number represent?

Acceleration due to gravity.