Read Section 3.6. Work the embedded problems.

1. Two Versions of the Chain Rule

$\boxed{A}$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

(In words) Take derivative of outside function w/ inside unchanged. Then multiply by derivative of inside function.

$\boxed{B}$ If $y = f(u)$ and $u = g(x)$ then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Leibniz notation

2. Use version $\boxed{B}$ to find $\frac{dy}{dx}$ if $y = 3\sqrt{u}$ $= 3u^{+1/2}$ and $u = \cos(x) + 1$.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(3 \cdot \frac{1}{2} u^{-\frac{1}{2}}\right)\left(-\sin(x) + 0\right)$$

$$= -\frac{3}{2}\left(\cos(x)+1\right)^{-1/2}\sin(x) = \frac{-3\sin(x)}{2\sqrt{\cos(x)+1}}$$

3. For each function below, decompose the function into the form $y = f(u)$ and $u = g(x)$ and then find $\frac{dy}{dx}$ using version $\boxed{B}$.

(a) $y = (3x - 5)^8$

$u = g(x) = 3x - 5$

$y = f(u) = 1 + u^8$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (0 + 8u^7)(3)$$

$$= 8(3x-5)^7(3)$$

$$= 24(3x-5)^7$$

(b) $y = \dfrac{1}{x^3 + \tan(x)}$

$u = g(x) = x^3 + \tan(x)$

$y = f(u) = \frac{1}{u} = u^{-1}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(-u^{-2}\right)\left(3x^2 + \sec^2(x)\right)$$

$$= -\left(x^3 + \tan(x)\right)^{-2}\left(3x^2 + \sec^2(x)\right)$$

$$= \frac{-\left(3x^2 + \sec^2(x)\right)}{\left(x^3 + \tan(x)\right)^2}$$

4. Find $\frac{dy}{dx}$ using version $\boxed{\text{A}}$.

(a) $y = (\frac{1}{x^2} + \frac{x^2}{3})^4$ $= (x^{-2} + \frac{1}{3}x^2)^4$

$y' = 4(x^{-2} + \frac{1}{3}x^2)^3 \cdot (-2x^{-3} + \frac{2}{3}x)$

(b) $y = \cos(2x)$

$y' = (-\sin(2x)) \cdot (2) = -2\sin(2x)$

(c) $y = \sqrt{x^2 + \sin(x)}$ $= (x^2 + \sin(x))^{1/2}$

$y' = \frac{1}{2}(x^2 + \sin(x))^{-1/2}(2x + \cos x) = \dfrac{2x + \cos x}{2\sqrt{x^2 + \sin(x)}}$

(d) $y = x\tan(\frac{\pi x}{4})$ $= x \cdot \tan(\frac{\pi}{4}x)$

product rule and chain rule

$y' = 1 \cdot \tan(\frac{\pi}{4}x) + x \cdot (\sec^2(\frac{\pi}{4}x))\frac{\pi}{4} = \tan(\frac{\pi}{4}x) + \frac{\pi}{4}x\sec^2(\frac{\pi}{4}x)$

(e) $y = \frac{x}{\sin^2(x)}$ $= \dfrac{x}{(\sin(x))^2}$ $= x(\sin x)^{-2}$ (prod + chain)

quotient + chain

$y' = \dfrac{(\sin(x))^2 \cdot 1 - x \cdot 2(\sin(x))^1\cos(x)}{(\sin x)^4}$

$= \dfrac{\sin(x)[\sin(x) - 2x\cos(x)]}{(\sin x)^4}$

$= \dfrac{\sin x - 2x\cos(x)}{(\sin(x))^3}$

$y' = 1 \cdot (\sin(x))^{-2} + x \cdot (-2)(\sin x)^{-3}(\cos(x))$

$= \dfrac{1}{(\sin x)^2} - \dfrac{2x\cos(x)}{(\sin(x))^3}$

$= \dfrac{\sin(x) - 2x\cos(x)}{(\sin(x))^3}$