1. Limits at Infinity: In plain English, what should the symbols below mean?

   $\lim_{x\to\infty} f(x) = L$ As x gets bigger + bigger, f(x) gets close to L.

   $\lim_{x\to-\infty} f(x) = L$ As x gets smaller + smaller, f(x) gets close to L.

2. Three Principles (a is a constant) and a Strategy

   - If a is a constant, then $\lim_{x\to\pm\infty} ax = \pm\infty$ (depending on the sign of a and x)
   - $\lim_{x\to\pm\infty} \frac{1}{x} = 0$
   - If $\lim_{x\to\pm\infty} f(x) = a$ and $\lim_{x\to\pm\infty} g(x) = \pm\infty$, then $\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} = 0$
   - Strategy: Divide numerator and denominator by the highest power of x in the denominator.

3. Use the Principles to evaluate the limits below. Then, use your calculator to confirm your answer is correct.

   (a) $\lim_{x\to\infty} \frac{(2x^2 - x)\cdot\frac{1}{x^2}}{(3x - 5x^2)\cdot\frac{1}{x^2}} = \lim_{x\to\infty} \frac{2 - \frac{1}{x}}{\frac{3}{x} - 5} = \frac{2}{-5} = -\frac{2}{5}$

   Check: $\frac{2(1000)^2 - (1000)}{3(1000) - 5(1000)^2} = -0.4000400...$ ✓

   (b) $\lim_{x\to\infty} \frac{(2x^3 - x)\cdot\frac{1}{x^2}}{(3x - 5x^2)\cdot\frac{1}{x^2}} = \lim_{x\to\infty} \frac{2x - \frac{1}{x}}{\frac{3}{x} - 5} = \infty$

   (c) $\lim_{x\to\infty} \frac{(3x + \sin(x))(\frac{1}{x})}{(x)(\frac{1}{x})} = \lim_{x\to\infty} \frac{3 + \frac{\sin(x)}{x}}{1} = 3$

   $\frac{\sin(x)}{x} \to 0$ b/c $-1 \le \sin(x) \le 1$

   (d) $\lim_{x\to-\infty} \frac{(2x+1)(\frac{1}{x})}{\sqrt{x^2+1}\,(\frac{1}{x})} = \lim_{x\to-\infty} \frac{2 + \frac{1}{x}}{-\sqrt{1 + \frac{1}{x^2}}} = \frac{2}{-1} = -2$

   thinking
   If $x \ge 0$, then $x = \sqrt{x^2}$
   If $x < 0$, then $x = -\sqrt{x^2}$

(many answers here...)

4. Construct a function $f(x)$ with a vertical asymptote at $x = 2$ and a horizontal asymptote at $x = 5$. Then **use limits** to demonstrate you are correct.

$$f(x) = \frac{5x}{x-2}$$

h.a.: $\lim\limits_{x\to\infty} \frac{5x}{x-2} = \lim\limits_{x\to\infty} \frac{5}{1-\frac{2}{x}} = 5$ ✓ $y=5$ is a horizontal asymptote

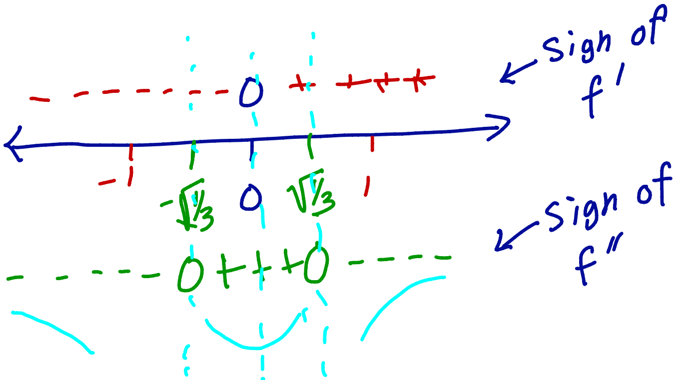
v.a.: $\lim\limits_{x\to 2^+} \frac{5x}{x-2} = +\infty$ b/c as $x\to 2^+$, $5x \to 10 > 0$ and $x-2 \to 0^+ > 0$.

5. Given $f(x) = \frac{x^2}{x^2+1}$, $f'(x) = \frac{2x}{(x^2+1)^2}$, $f''(x) = \frac{-2(3x^2-1)}{(x^2+1)^3}$. Identify important features of $f(x)$ like: asymptotes, local extrema, inflection points, and make a rough sketch.

asymptotes: v.a. none; h.a at $y=1$ $\left(\lim\limits_{x\to\pm\infty} \frac{x^2}{x^2+1} = 1\right)$

$\uparrow, \downarrow$, extrema: $f'(x) = 0$ when $x=0$.

$f(x)$ is $\downarrow$ on $(-\infty, 0)$ and $\uparrow$ on $(0,\infty)$.
$f(x)$ has an absolute min at $x=0$.
and no local or abs max.



concavity $f''(x) = 0$ when

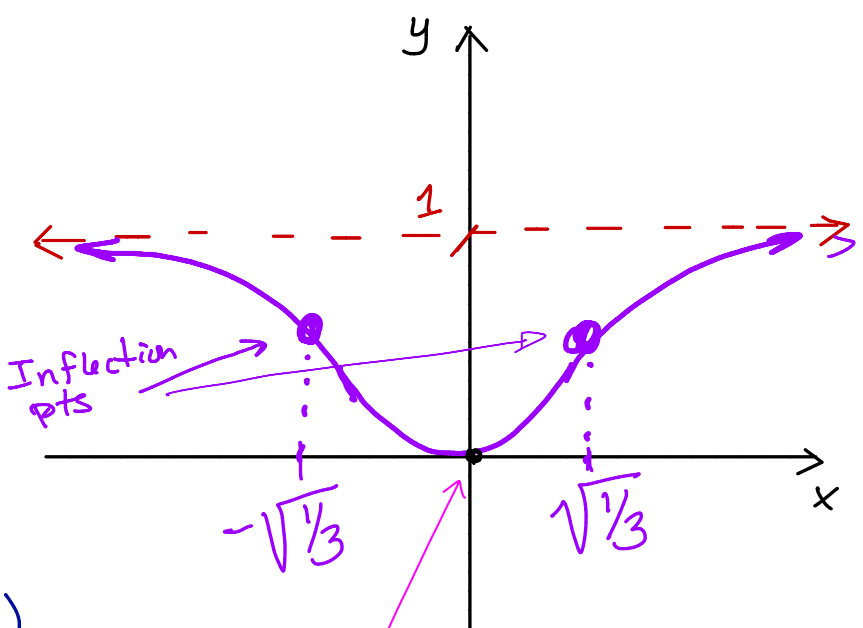
$3x^2 - 1 = 0$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{1/3}$

$f(x)$ is cc up on $(-\sqrt{1/3}, \sqrt{1/3})$

and cc down on $(-\infty, -\sqrt{1/3}) \cup (\sqrt{1/3}, \infty)$



absolute minimum of 0 at $x=0$