SECTION 4.6: LIMITS AT INFINITY AND ASYMPTOTES

1. Limits at Infinity: In plain English, what should the symbols below mean?

   $\lim_{x\to\infty} f(x) = L$

   $\lim_{x\to-\infty} f(x) = L$

2. Three Principles (a is a constant) and a Strategy

   - If a is a constant, then $\lim_{x\to\pm\infty} ax =$
   - $\lim_{x\to\pm\infty} \frac{1}{x} =$
   - If $\lim_{x\to\pm\infty} f(x) = a$ and $\lim_{x\to\pm\infty} g(x) = \pm\infty$, then $\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} =$
   - Strategy: Divide numerator and denominator by the highest power of x in the denominator.

3. Use the Principles to evaluate the limits below. Then, use your calculator to confirm your answer is correct.

   (a) $\lim_{x\to\infty} \frac{2x^2 - x}{3x - 5x^2}$

   (b) $\lim_{x\to\infty} \frac{2x^3 - x}{3x - 5x^2}$

   (c) $\lim_{x\to\infty} \frac{3x + \sin(x)}{x}$

   (d) $\lim_{x\to-\infty} \frac{2x + 1}{\sqrt{x^2 + 1}}$

4. Construct a function $f(x)$ with a vertical asymptote at $x = 2$ and a horizontal asymptote at $x = 5$. Then **use limits** to demonstrate you are correct.

5. Given $f(x) = \frac{x^2}{x^2+1}$, $f'(x) = \frac{2x}{(x^2+1)^2}$, $f''(x) = \frac{-2(3x^2-1)}{(x^2+1)^3}$. Identify important features of $f(x)$ like: asymptotes, local extrema, inflection points, and make a rough sketch.