SECTION 4.7 OPTIMIZATION (DAY 1)

1. A Framework for Approaching Optimization

   (a) **Identify** the quantity to be minimized or maximized.

   (b) Chose **notation** and explain what it means.

   (c) Write the thing you want to maximize or minimize **as a function of one variable**, including a reasonable **domain**.

   (d) Use **calculus** to answer the question and **justify** that your answer is correct.

   Read the problem two or three times. Draw pictures. Label them. Pick specific numerical examples, to make the problem concrete. Be creative. Try more than just one approach. Organization matters.

2. Find two positive numbers whose sum is 123 and whose product is a maximum.

a. maximize product maximize $xy = P =$ product

b. x, y - two positive numbers. so

c. need P as fcn of x or y, not both. Use $x+y=123$; So $y=123-x$

Plug into P: $P = xy = x(123-x) = 123x - x^2$

Now: $\boxed{P(x) = 123x - x^2}$ ← Do you see why P is replaced w/ $P(x)$?!

Domain: $\boxed{x>0}$ b/c positive number. or domain $P(x)$ is $(0,\infty)$

d. Use Calc to find max:

$P(x) = 123x - x^2$ on $(0,\infty)$

(Find c.p.:) $P'(x) = 123 - 2x$ $P' = 0$ when $\boxed{x = \frac{123}{2} = 62.5}$

P' is never undefined!

(Use 2^{nd} Der Test:) $P''(x) = -2 < 0$ for all x. (so P is ccdown)

max ✓

c

(Justification:) By the 2^{rd} Der Test, P has a local max at $x = 62.5$. It is an absolute max because $x=62.5$ is the only critical point.

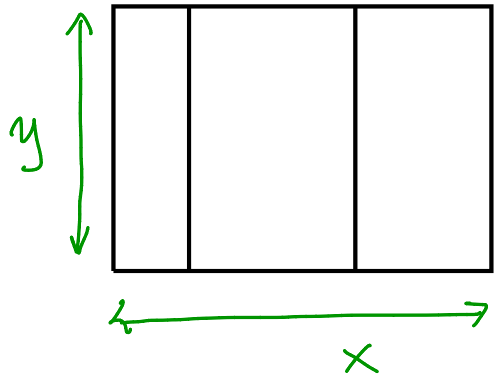
ANSWER: The positive numbers that maximize the product are $x = y = 62.5$.

Note: You could use 1'st derivative test: ← sign P'

$+ + 0 - - -$

0 62.5 ∞

3. A rancher has 1255 feet of fencing with which to enclose three adjacent rectangular corrals. See figure below. What dimensions should be used so that the enclosed area will be a maximum?



a. maximize total enclosed area.

$A = xy$

b. label picture (see pic) So

c. Need A as a function of x or y, not both.

Use $1255 = \underbrace{2x}_{\text{top+bottom}} + \underbrace{4y}_{\text{sides+partitions}}$

So $x = \dfrac{1255 - 4y}{2} = \dfrac{1255}{2} - 2y$. Plug into A: $A = xy = \left(\dfrac{1255}{2} - 2y\right)(y)$

So $\boxed{A(y) = \dfrac{1255}{2}y - 2y^2.}$ Domain: $\boxed{y \geq 0.}$ (y = length so it can't be −)

d. Use Calculus:

(Find crit. pts:) $A'(y) = \dfrac{1255}{2} - 4y = 0$, $4y = \dfrac{1255}{2}$ or $y = \dfrac{1255}{8}$.

$y = \dfrac{1255}{8}$ is the only crit. pt. since $A'(y)$ is never undefined.

(Justify:) $A(y)$ is a parabola that opens down, so $y = \dfrac{1255}{8}$ must correspond to an abs. max.

Answer: The dimensions of the pen, in feet, are

$y = \dfrac{1255}{8}$ and $x = \dfrac{1255}{2} - \dfrac{1255}{4} = \dfrac{1255}{4}$.