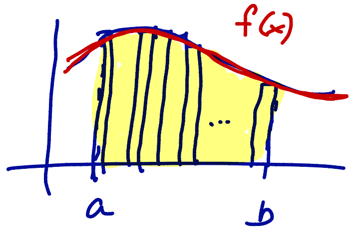
1. **Definition of the Definite Integral:** (abbreviated)

$$* \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\underset{\text{height}}{f(x_i)} \cdot \underset{\text{width}}{w_i}}_{\text{area}} = \text{net signed area} = \left(\begin{array}{c}\text{area}\\ \text{above}\\ x\text{-axis}\end{array}\right) - \left(\begin{array}{c}\text{area}\\ \text{below}\\ x\text{-axis}\end{array}\right)$$



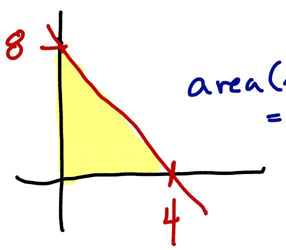

$*$ Compare to $\int f(x)\,dx = \boxed{F(x)+C}$ ← a function.

$\int_a^b f(x)\,dx = \#$ ← a number

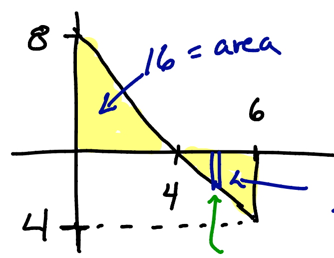
2. Evaluate the definite integrals below using the graph and geometry.

(a) $\displaystyle\int_0^4 (8-2x)\,dx = \frac{1}{2}(4)(8) = 16$

(b) $\displaystyle\int_0^6 (8-2x)\,dx = 16 - 4 = 12$




area($\Delta$) $= \frac{1}{2}$(base)(height)



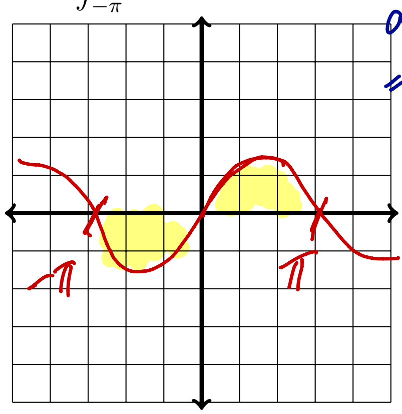

area $\frac{1}{2} \cdot 2 \cdot 4 = 4$

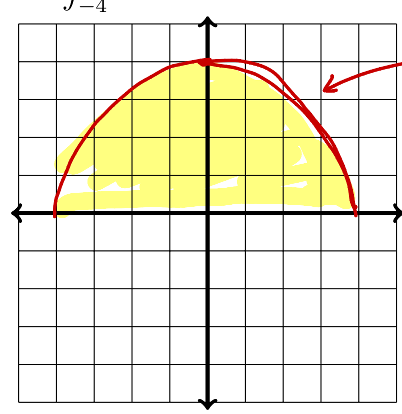
but the height, given by $f(x)$, is negative.

3. Evaluate the following definite integrals by drawing the function and interpreting the integral in terms of areas. Shade in the area you are computing with the integral.

(a) $\displaystyle\int_{-\pi}^{\pi} \sin(x)\,dx = \underline{\;0\;}$ ← b/c area above = area below

(b) $\displaystyle\int_{-4}^{4} \sqrt{16-x^2}\,dx = \underline{\tfrac{1}{2}\pi \cdot 4^2} = 8\pi$



circle, top-half, radius = 4

4. Definition: average value of a function over the interval $[a,b]$:

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

think:

$$(f_{ave})(b-a) = \int_a^b f(x)\,dx$$

$f_{ave}$: height of rectangle w/ area $\int_a^b f(x)\,dx$

$*$ What is the average value of $f(x) = 8-2x$ on $[0,4]$?

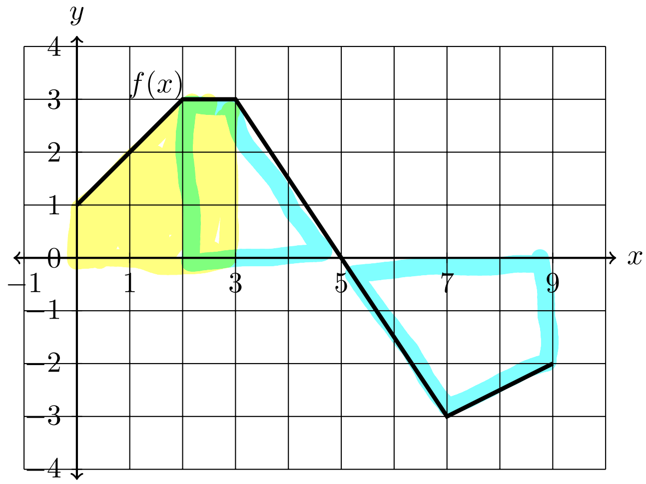
$$f_{ave} = \frac{1}{4}\int_0^4 (8-2x)\,dx = \frac{16}{4} = 4$$

**Properties of the Definite Integral:**

- $\int_a^b f(x)\,dx =$ net signed area!
- $\int_a^a f(x)\,dx =$ 0
- $\int_a^b c\,dx =$ $c(b-a)$
- $\int_a^b cf(x)\,dx =$ $c\int_a^b f(x)dx$

  * take a constant outside
- $\int_a^b [f(x) \pm g(x)]dx =$ $\int_a^b f(x)dx \pm \int_a^b g(x)dx$
- $\int_a^b f(x) + \int_b^c f(x)\,dx =$ $\int_a^c f(x)dx$
- $\int_b^a f(x)\,dx =$ $-\int_a^b f(x)dx$

5. The graph of $f$ is shown. Evaluate each integral by interpreting it in terms of areas.




(a) $\int_0^3 8f(x)dx =$ $8\int_0^3 f(x)dx = 8\cdot 7$
$= 56$

(b) $\int_2^9 f(x)dx =$ $3+3-3-5=-2$

(c) $\int_5^3 f(x)dx =$ $-3$

6. Using the fact that $\int_0^1 x^2dx = \frac{1}{3}$ and $\int_1^2 x^2\,dx = \frac{7}{3}$, evaluate the following using the properties of integrals.

(a) $\int_0^1 5x^2\,dx$

$= 5\int_0^1 x^2 dx = 5\cdot\frac{1}{3}$
$= \frac{5}{3}$

(b) $\int_0^1 (4+3x^2)\,dx$

$= \int_0^1 4\,dx + 3\int_0^1 x^2 dx$
$= 4\cdot 1 + 3\cdot\frac{1}{3}$
$= 5$

(c) $\int_0^2 x^2\,dx.$

$= \int_0^1 x^2 dx + \int_1^2 x^2 dx$
$= \frac{1}{3} + \frac{7}{3} = \frac{8}{3}$

$f_{ave}$ on $[0,2]$?

$\frac{1}{2}\int_0^2 x^2 dx = \frac{1}{2}\cdot\frac{8}{3} = \frac{4}{3}$