

SECTION 5.4: THE NET CHANGE THEOREM

1. Quick Review: Evaluate the following.

(a) $\int \left(\frac{x}{3} - \sin(x) \right) dx$

(b) $\int_0^5 (3 - e^x) dx$

(c) $\frac{d}{dx} \left(\int_1^{x^2} (\ln(t)) dt \right)$

2. Assume $P'(t)$ gives the rate of change in a population of ants over time where time t is measured in days and $P(t)$ is measured in hundreds of ants per day. Use the table below to answer the questions.

t	0	7	14	21	28	35
$P'(t)$	0	1.9	2.4	2.7	3.0	3.2

(a) Interpret $P'(14) = 2.4$.

(b) Estimate how much the ant population increased in the first three weeks. Include units with your answer.

(c) What would $\int_0^{21} P'(t) dt$ represent?

(d) What would $P(t)$ represent? What is $P(14)$?

3. The Net Change Theorem:

4. Let $w'(t)$ be the rate of growth of a child in pounds per year.

(a) What does $\int_5^{10} w'(t) dt$ represent? (Write a complete sentence a regular person could understand.)

(b) Explain what $w(10)$ represents.

5. Snow is falling on my garden at a rate of $m'(t) = 6t$ kilograms per hour for $0 \leq t \leq 2$ where t is measured in hours.

(a) Find and interpret $m'(1)$.

(b) Find an interpret $\int_0^2 m'(t) dt$

(c) In this context, what would $m(0) = 13$ represent?

(d) Find and interpret $m(2)$.

6. The height of water in a cylindrical tank is modeled by $h'(t) = 3 \sin(t)$ where h' is measured in meters per hour and t is measured in hours. It is a fact that

$$\int_0^\pi h'(t)dt = 6 \text{ and } \int_\pi^{2\pi} h'(t)dt = -6.$$

(You should check this on your own.)

Use the information to find $\int_0^{2\pi} h'(t)dt$. Can you explain what is happening in this tank? Do you think the tank is running out of water?