Section 5.6: Integrals Involving Exponentials and Logarithmic Functions

1. On Monday, we started integrating using the Method of Substitution. Describe in words (and examples if you like) *how* we figured out what to pick to be u when using this method?

Pick u to be

- something raised to a power: u^6 or $\frac{1}{\sqrt{u}}$

or

- pick u to be inside a trig function: $\cos(u)$

2. Complete the integration formulas below:

(a) $\int e^x\,dx = e^x + C$

(b) $\int a^x\,dx = \frac{a^x}{\ln a} + C$

(c) $\int \frac{1}{x}\,dx = \ln|x| + C$

(d) $\int \ln(x)\,dx = x\ln(x) - x + C$

$= x(\ln(x) - 1) + C$

(e) $\int \log_a(x)\,dx = \frac{1}{\ln a}\left(x\ln(x) - x\right) + C$

use $\log_a x = \frac{\ln x}{\ln a}$

3. Examples to illustrate four more standard ways to select u.

(a) $\int xe^{x^2}\,dx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$

let $u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

(b) $\int \frac{x^2}{x^3-7}\,dx = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3-7| + C$

let $u = x^3 - 7$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

(c) $\int 3x\ln(10+x^2)\,dx = \frac{3}{2}\int \ln u\,du = \frac{3}{2}u(\ln u - 1) + C$

$= \frac{3}{2}(10+x^2)(\ln(10+x^2) - 1) + C$

let $u = 10 + x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

(d) $\int \frac{\ln(x)}{x}\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$

let $u = \ln(x)$

$du = \frac{1}{x}dx$

4. Evaluate the integrals below. Be creative!

(a) $\int_2^3 \frac{1}{x\ln(x)}\,dx = \int_{\ln 2}^{\ln 3} \frac{1}{u}\,du = \ln(u)\Big]_{\ln 2}^{\ln 3} = \ln(\ln(3)) - \ln(\ln(2))$

let $u = \ln(x)$

$du = \frac{1}{x}\,dx$

If $x = 2$, $u = \ln 2$

If $x = 3$, $u = \ln 3$

(b) $\int_1^4 \frac{5}{\sqrt{x}e^{\sqrt{x}}}\,dx = 5\int_1^4 e^{-x^{1/2}} \cdot \frac{dx}{\sqrt{x}} = 5(-2)\int_{-1}^{-2} e^u\,du = -10e^u\Big]_{-1}^{-2}$

let $u = -x^{1/2}$

$du = -\frac{1}{2}x^{-1/2}\,dx$

$-2\,du = \frac{dx}{\sqrt{x}}$

if $x=1$, $u = -\sqrt{1} = -1$

if $x=4$, $u = -\sqrt{4} = -2$

$= -10(e^{-2} - e^{-1}) = 10\left(\frac{1}{e} - \frac{1}{e^2}\right)$

(c) $\int_0^{\pi/4} \tan(x)\,dx = \int_0^{\pi/4} \frac{\sin(x)\,dx}{\cos(x)} = -\int_1^{\sqrt{2}/2} u^{-1}\,du = -\ln u\Big]_1^{\sqrt{2}/2}$

let $u = \cos(x)$

$du = -\sin(x)\,dx$

$-du = \sin(x)\,dx$

if $x=0$, $u=1$;

if $x = \pi/4$, $u = \cos(\pi/4) = \frac{\sqrt{2}}{2}$

$= -\ln(1) - \left(-\ln\left(\frac{\sqrt{2}}{2}\right)\right)$

$= \boxed{\ln\left(\sqrt{2}/2\right)}$

(d) $\int \ln(\cos(x))\tan(x)\,dx = \int \frac{\ln(\cos(x))\sin(x)\,dx}{\cos(x)}$

let $u = \ln(\cos(x))$

$du = \frac{1}{\cos x}(-\sin(x))\,dx$

$= -\int u\,du = -\frac{1}{2}u^2 + C = \frac{-1}{2}(\ln(\cos(x)))^2 + C$

(e) $\int \frac{e^{4x} - e^{-4x}}{e^{4x} + e^{-4x}}\,dx = \frac{1}{4}\int u^{-1}\,du = \frac{1}{4}\ln(u) + C$

$= \boxed{\frac{1}{4}\ln(e^{4x} + e^{-4x}) + C}$

let $u = e^{4x} + e^{-4x}$

$du = 4e^{4x} - 4e^{-4x}\,dx$

$\frac{1}{4}\,du = e^{4x} - e^{-4x}\,dx$