1. Describe in words (and examples if you like) different strategies for picking the u in the method of integration called "Substitution."

- something raised to a power $(u)^p$
- under a radical $\sqrt{u}$ or $\frac{1}{\sqrt{u}}$
- the denominator $\frac{1}{u}$
- the exponent of e: e^u
- inside a trig fch: $\sin(u)$
- inside a function $\ln(u)$
- get creative, try stuff

2. Determine the Integral Formulas the result from that derivatives of inverse sine and inverse tangent.

(a) $\displaystyle\int \frac{dx}{\sqrt{1-x^2}} = \arcsin(x) + c$

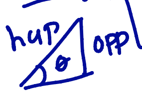
(b) $\displaystyle\int \frac{dx}{1+x^2} = \arctan x + c$

3. Some simple examples (+ some trig)

(a) $\displaystyle\int_0^{\sqrt{2}/2} \frac{dx}{\sqrt{1-x^2}} = \arcsin(x)\Big]_0^{\sqrt{2}/2} = \arcsin\left(\sqrt{2}/2\right) - \arcsin(0) = \frac{\pi}{4}$

angle $\theta \rightarrow \boxed{\sin(\theta)} \rightarrow \frac{opp}{hyp}$

opp/hyp $\rightarrow \boxed{\arcsin\left(\frac{opp}{hyp}\right)} \rightarrow \theta$



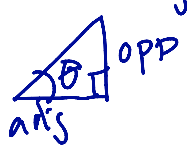
$\sin(0)=0$ means $\arcsin(0)=0$

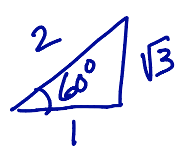
(b) $\displaystyle\int_1^{\sqrt{3}} \frac{2dx}{1+x^2} = 2(\arctan x)\Big]_1^{\sqrt{3}} = 2\left[\arctan\sqrt{3} - \arctan 1\right] = 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \pi/6$

$\tan\theta = \frac{opp}{adj}$

$\arctan\left(\frac{opp}{adj}\right) = \theta$





$60 = \frac{\pi}{3}$

4. The algebra required to remember nothing more than the formulas on page 1.

(a) $\int \frac{dx}{1+5x^2} = \int \frac{dx}{1+(\sqrt{5}x)^2} = \frac{1}{\sqrt{5}}\int \frac{du}{1+u^2} = \frac{1}{\sqrt{5}} \arctan(u) + C$

Let $u = \sqrt{5}x$

$du = \sqrt{5}\,dx$

$\frac{1}{\sqrt{5}}du = dx$

$= \frac{1}{\sqrt{5}} \arctan(\sqrt{5}x) + C$

(b) $\int \frac{dx}{5+x^2} = \frac{1}{5}\int \frac{dx}{1+\left(\frac{x}{\sqrt{5}}\right)^2} = \frac{\sqrt{5}}{5}\int \frac{du}{1+u^2} = \frac{\sqrt{5}}{5}\arctan(u) + C$

$= \frac{\sqrt{5}}{5}\arctan\left(\frac{x}{\sqrt{5}}\right) + C$

$5+x^2$

$= 5\left(1+\frac{x^2}{5}\right)$

$= 5\left(1+\left(\frac{x}{\sqrt{5}}\right)^2\right)$

Let $u = x/\sqrt{5}$

$du = \frac{1}{\sqrt{5}}dx$

$\sqrt{5}\,du = dx$

(c) $\int \frac{7dx}{4+3x^2} = \frac{7}{4}\int \frac{dx}{1+\left(\frac{\sqrt{3}x}{2}\right)^2} = \frac{7}{4}\cdot\frac{2}{\sqrt{3}}\int \frac{du}{1+u^2} = \frac{7}{2\sqrt{3}} \arctan u + C$

$4+3x^2$

$= 4\left(1+\frac{3}{4}x^2\right)$

$= 4\left(1+\left(\frac{\sqrt{3}x}{2}\right)^2\right)$

Let $u = \frac{\sqrt{3}x}{2}$

$\frac{2}{\sqrt{3}}du = dx$

$= \frac{7}{2\sqrt{3}} \arctan\left(\frac{\sqrt{3}}{2}x\right) + C$

5. You evaluate $\int \frac{dx}{\sqrt{1+\frac{x^2}{2}}} = \int \frac{dx}{\sqrt{1+\left(\frac{x}{\sqrt{2}}\right)^2}} = \sqrt{2}\int \frac{du}{\sqrt{1-u^2}} = \sqrt{2}\sin\left(\frac{x}{\sqrt{2}}\right) + C$

$u = \frac{x}{\sqrt{2}}$

$\sqrt{2}\,du = dx$

$7\int \frac{\frac{1}{\sqrt{3}}\sqrt{3}\,dx}{2^2+(\sqrt{3}x)^2} = \frac{7}{\sqrt{3}}\int \frac{\sqrt{3}\,dx}{2^2+(\sqrt{3}x)^2}$

$= \frac{7}{\sqrt{3}}\cdot\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + C$

$a^2 = 4,\ a = 2$

$u^2 = 3x^2 = (\sqrt{3}x)^2$

$u = \sqrt{3}x;\ du = \sqrt{3}\,dx$

6. The fancy formulas.

(a) $\int \frac{du}{\sqrt{u^2-a^2}} = \frac{1}{|a|}\sec^{-1}\left(\frac{u}{a}\right) + C$

(c) $\int \frac{du}{\sqrt{a^2+u^2}} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

(b) $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$