WORKSHEET: REVIEW OF FUNCTIONS

Goals:

- How to think about and use function notation and terminology.
- A list of functions to know.
- Some practice putting these together.

1. The notation $y = f(x)$ means • "y is a function of x" or "y-values are determined by x-values"

• for every reasonable x-value, $f(x)$ will give exactly 1 y-value.

• $x \to \boxed{f} \to y$ (black box view)

• It's graph passes the vertical line test. [Every vertical line intersects the graph at most 1 time]

2. Let $f(x) = 10 - 3x^2$. Find and simplify the following expressions.

(a) $f(5) = 10 - 3(5)^2$

$= 10 - 3 \cdot 25$

$= 10 - 75$

$= -65$

(b) $f(3a) = 10 - 3(3a)^2$

$= 10 - 3(27a^2)$

$= 10 - 81a^2$

(c) $2\,[f(a)]^2$

aside: $f(a) = 10 - 3a^2$

So $(f(a))^2 = (10 - 3a^2)^2 = 100 - 60a^2 + 9a^4$

Finally, $2[f(a)]^2 = 2[100 - 60a^2 + 9a^4] = 200 - 120a^2 + 18a^4$

(d) $f(x+h) = 10 - 3(x+h)^2$

$= 10 - 3(x^2 + 2xh + h^2)$

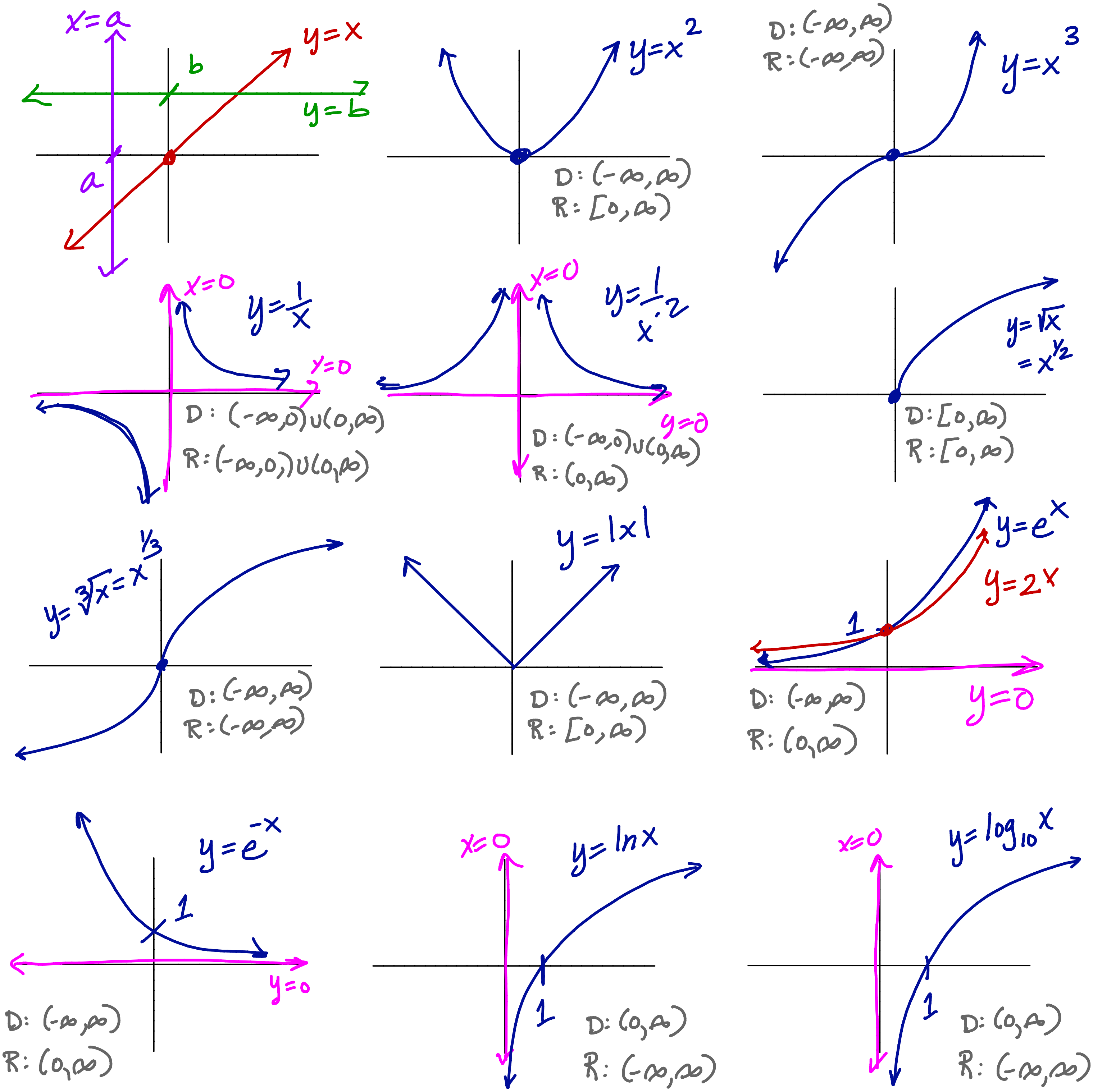
$= 10 - 3x^2 - 6xh - 3h^2$

(e) $f(x) + h = 10 - 3x^2 + h$

always in Pink!

3. Below is a list of expressions you should be able to graph instantly. Your graphs should always include any x- and y-intercepts, asymptotes, and clearly indicate end behavior.

$$y = x,\quad y = b,\quad x = a,\quad y = x^2,\quad y = x^3,\quad y = \frac{1}{x},\quad y = \frac{1}{x^2},\quad y = \sqrt{x},\quad y = \sqrt[3]{x}$$
$$y = |x|,\quad y = e^x,\quad y = 2^x,\quad y = e^{-x},\quad y = \ln x,\quad y = \log_{10}(x)$$



Include domain and range! In gray

4. Write the equation of the line through the point $(2, -5)$ that is parallel to the line $4x + 3y = 17$.

- to write equation, need slope (m) and point (x_0, y_0).
- Point: $(2,-5)$
- Slope: Put $4x+3y=17$ into slope-intercept form + find slope!

$$y = -\frac{4}{3}x + \frac{17}{3}. \quad \text{So } m = -\frac{4}{3}$$

- Use point-slope form of line + plug in: $y - y_0 = m(x - x_0)$
- Answer: $y-(-5) = -\frac{4}{3}(x-2)$ or $y = -5 - \frac{4}{3}(x-2)$ or

$$y+5 = -\frac{4}{3}x + \frac{8}{3} \quad \text{or} \quad y = -\frac{4}{3}x - \frac{7}{3}$$

5. Find the domain and range of $f(x) = 4 + \sqrt{11 - x}$. Give your answers in interval notation. Explain how you got your answer.

For $f(x)$ to make sense, we need $11 - x \geq 0$. So $11 \geq x$.

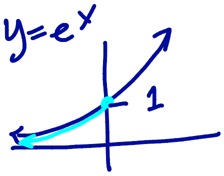
So domain D: $(-\infty, 11]$.

To find the range, we observe that the "$11-x$" part is a horizontal shift + reflection of $\sqrt{x}$. So it will not change the range. But the "$+4$" is a vertical shift 4-units up.

So range: $[4, \infty)$.

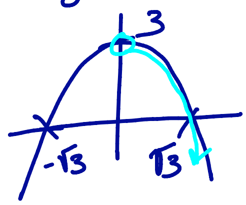
6. Sketch the graph of $f(x) = \begin{cases} e^x & x \le 0 \\ 3 - x^2 & 0 < x \end{cases}$

aside
$y = e^x$



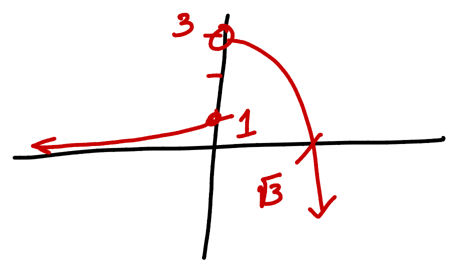
$x \le 0$

$y = 3 - x^2$



$x > 0$

Answer:



7. Determine any x- or y-intercepts for the graphs below.

(a) $g(x) = 2x^2 + 13x - 7$

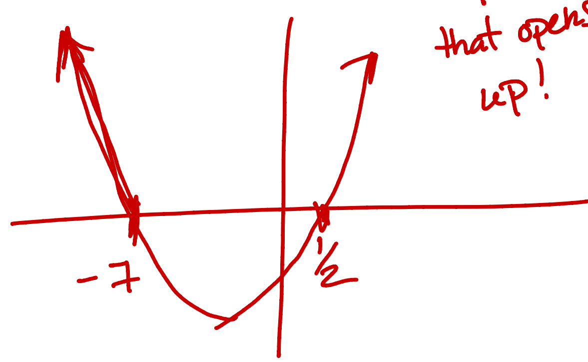
y-int: Set $x=0$. $g(0) = \underline{\underline{-7}}$

x-int: Set $y=0$.

$0 = 2x^2 + 13x - 7 = (2x-1)(x+7)$

So $2x - 1 = 0$ or $x + 7 = 0$

So $x = \underline{\underline{\tfrac{1}{2}}}$ or $x = \underline{\underline{-7}}$



A parabola that opens up!

Answer: y-int at $y=-7$, x-int at $x=\frac{1}{2}$ and $x=-7$.

(b) $h(x) = \frac{a}{x-b}$ (Assume a and b are fixed positive constants.)

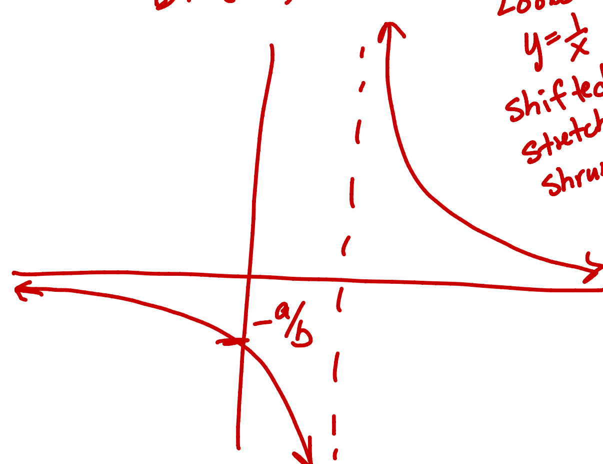
y int: Set $x=0$. $h(0) = \frac{a}{0-b} = -\frac{a}{b}$

x-int: Set $y=0$. $0 = \frac{a}{x-b}$ (has no solution)

none!

Answer: y-int when $y = -a/b$.
no x-intercepts.

D: $(-\infty, b) \cup (b, \infty)$



Looks like $y = \frac{1}{x}$ but shifted + stretched/shrunk

8. Bonus: Sketch the functions g and h from the previous problem.