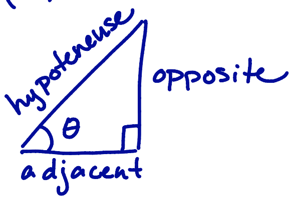
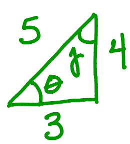
1. There are three particularly useful ways of thinking about trigonometric functions: (A) sides of a right triangle, (B) points on the unit circle in the xy-plane, (C) as a graph. Can you describe the sine function in each of these ways?

(A) Sides of a right triangle



$\sin(\theta) = \frac{opp}{hyp}$

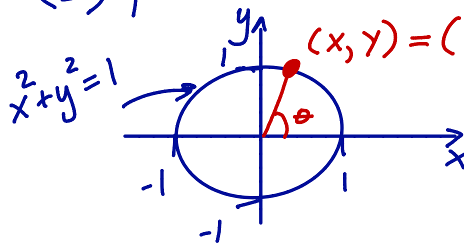
Ex



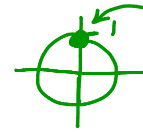
$\sin(\theta) = \frac{4}{5}$

$\sin(\gamma) = \frac{3}{5}$

(B) points on unit circle



Ex| Find θ where $\sin(\theta) = 1$. (where $y=1$)



So $\theta = 90° = \frac{\pi}{2}$

or $\theta = 360°+90° = 2\pi + \frac{\pi}{2}$

Answer: Any $\theta = 2\pi k + \frac{\pi}{2}$ for $k = \ldots -2, -1, 0, 1, 2, \ldots$

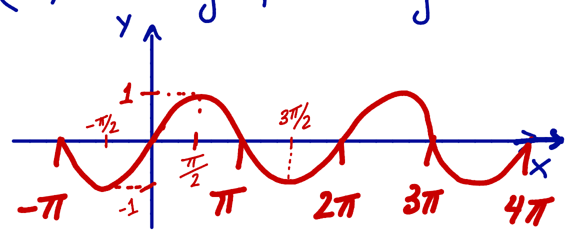
Other trig fcns:

$\cos\theta = \frac{adj}{hyp}$, $\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$

$\sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta}$, $\csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta}$

$\cot(\theta) = \frac{\cos\theta}{\sin\theta} = \frac{adj}{opp} = \frac{1}{\tan\theta}$

(C) The graph of $y = \sin(x)$.

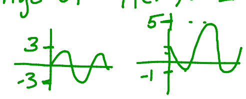


Ex| What is the domain + range of $H(x) = 2 + 3\sin(x)$.

thinking: $H(x) = 2 + 3\sin(x)$

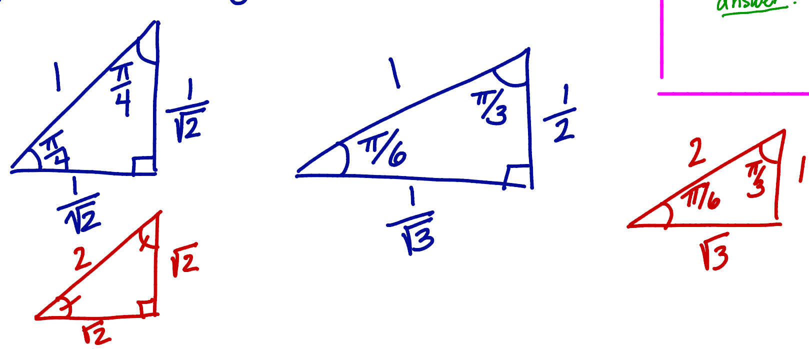
shifts up 2

changes amplitude



answer: D: $(-\infty, \infty)$ R: $[-1, 5]$

Special Triangles:

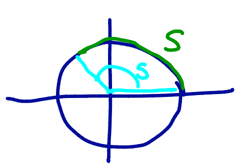


$\frac{\pi}{4}$ radians $= 45°$, $\frac{\pi}{6}$ rad $= 30°$, $\frac{\pi}{3} = 60°$

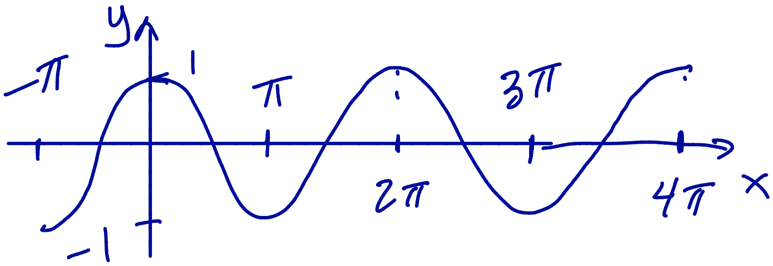
What is a radian?

radian is the angle so that angle # = arc length in unit circle.

π radians $= 180°$

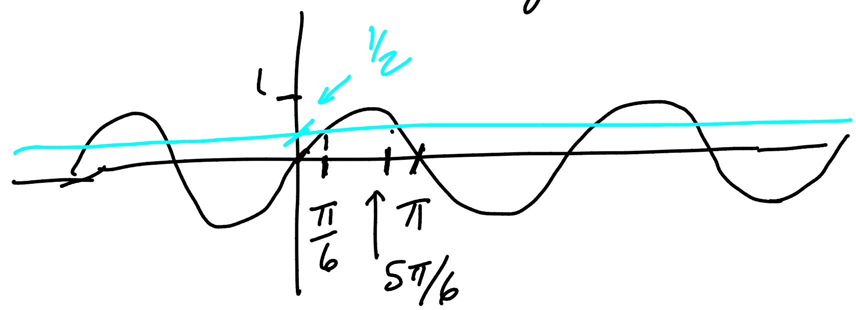


2. Sketch the graph of $f(x) = \cos(x)$ from $[-\pi, 4\pi]$ and the graph of $g(x) = \tan(x)$ from $[-\pi/2, \frac{3\pi}{2}]$.

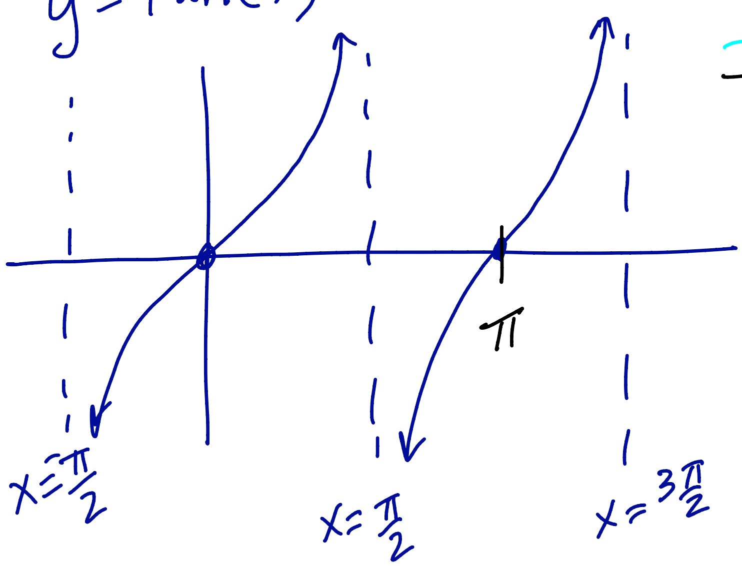


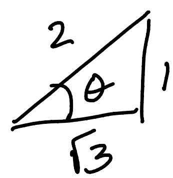
$y = \cos(x)$

$y = \sin x$



$y = \tan(x)$





$\theta = \frac{\pi}{6}$

3. Use graphs to solve the equations below.

(a) $\cos x = 1$

$x = \ldots -2\pi, 0, 2\pi, 4\pi, \ldots$

(b) $\sin x = 1$

$x = \ldots -2\pi + \frac{\pi}{2}, \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, \ldots$

(c) $\tan x = 0$

$x = \ldots, -\pi, 0, \pi, 2\pi, \ldots$

(d) $\sin x = 1/2$ (Find all solutions in $[0, 2\pi]$.)

$x = \ldots, -2\pi + \frac{\pi}{6}, \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, 4\pi + \frac{\pi}{6}, \ldots$

and

$x = \ldots, -2\pi + \frac{5\pi}{6}, \frac{5\pi}{6}, 2\pi + \frac{5\pi}{6}, \ldots$

4. Convert $2\pi/3$ radians and $5\pi/7$ radians to degrees.

$$\frac{2\pi}{3}\text{rad} = 2\left(\frac{\pi}{3}\text{rad}\right) = 2\cdot 60^\circ = 120^\circ$$

$$\left(\frac{5\cancel{\pi}}{7}\,\cancel{\text{rad}}\right)\left(\frac{180^\circ}{\cancel{\pi}\,\cancel{\text{rad}}}\right) = \frac{5(180)}{7} = \frac{900}{7}$$

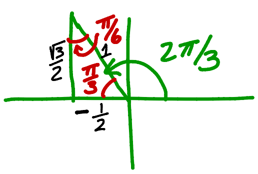
5. Convert 20 degrees to radians.

$$(20^\circ)\left(\frac{\pi\ \text{rad}}{180^\circ}\right) = \frac{20\pi}{180} = \frac{\pi}{9}\ \text{rad}$$

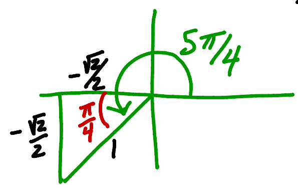
Method ? → memorize unit circle (See last page) → memorize important triangles (See first page.)

6. Without a calculator evaluate:

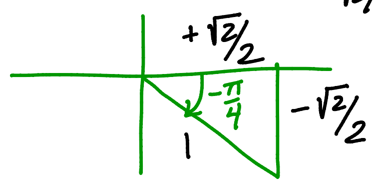
(a) $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$



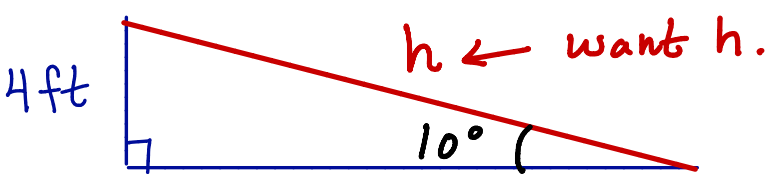
(b) $\cos(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2}$



(c) $\tan(\frac{-\pi}{4}) = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$



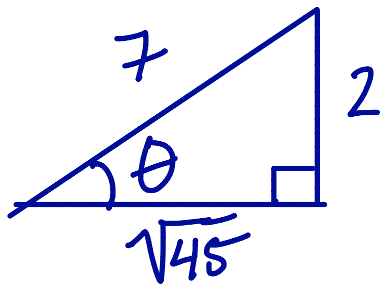
7. A wooden ramp is to be built with one end on the ground and the other end at the top of a short staircase. If the top of the staircase is 4 ft from the ground and the angle between the ground and the ramp is to be 10^o, how long does the ramp need to be?



So $\sin(10^\circ) = \frac{4}{h}$ or $h = \frac{4\text{ft}}{\sin(10^\circ)} = 23\text{ft}$

make sure you use the correct units in your calculator!

8. Find $\cos\theta$ assuming that $\sin\theta = 2/7$ and θ is in the first quadrant.



$$2^2 + x^2 = 7^2$$

$$x = \sqrt{49-4} = \sqrt{45}$$

$$\cos(\theta) = \sqrt{45}/7$$

9. Fill out the unit circle below without the use of a calculator.