SECTION 2-2: THE LIMIT OF A FUNCTION

Read Section 2.2. Work the embedded problems.
Goals:

- Understand the meaning of the notation $\lim_{x \to a} f(x) = L$.
- Know how to evaluate one- and two-sided limits both from a graph and numerically.
- Understand the relationship between infinite limits and vertical asymptotes.

1. DEFINITION: **two-sided limit**

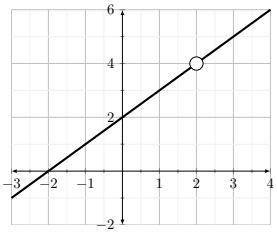
   Notation:

   Words:

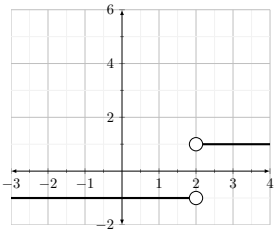
   It means:

2. Evaluate the limits below using the graph and confirm your answers numerically.

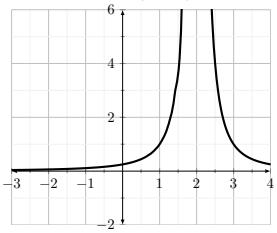
   (a) $f(x) = \frac{x^2-4}{x-2}$

   (b) $f(x) = \frac{|x-2|}{x-2}$

   (c) $f(x) = \frac{1}{(x-2)^2}$

   graphically:

   $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2} =$ $\lim_{x \to 2} \dfrac{|x - 2|}{x - 2} =$ $\lim_{x \to 2} \dfrac{1}{(x - 2)^2} =$

   numerically:

3. Numerically or graphically, determine the limits below. Assume a and c are fixed constants.

(a) $\lim_{x \to 0} = \dfrac{\sin(x)}{x}$

(b) $\lim_{x \to 1} 5 =$

(c) $\lim_{x \to 2} 5 =$

(d) $\lim_{x \to a} c =$

(e) $\lim_{x \to 1} x =$

(f) $\lim_{x \to 2} x =$

(g) $\lim_{x \to a} x =$

4. Return to problem 2b above. Evaluate the limits below assuming that

$x \to 2^-$ means

and

$x \to 2^+$ means

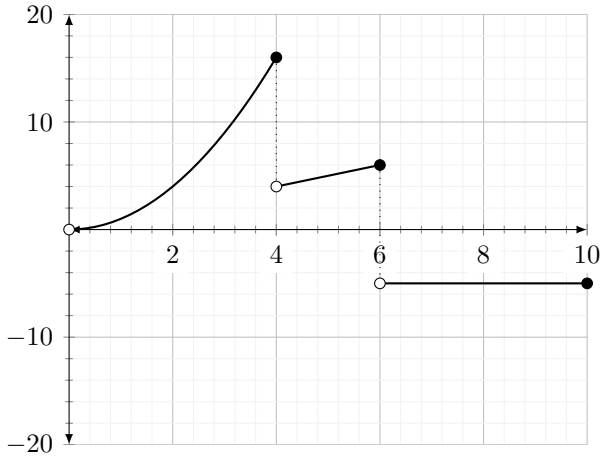
(a) $\lim_{x \to 2^-} \dfrac{|x-2|}{x-2} =$

(b) $\lim_{x \to 2^+} \dfrac{|x-2|}{x-2} =$

5. What must be the relationship between the existence of two-sided limits in terms of one-sided limits?

6. DEFINITION: infinite limits

7. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x \to 4^-} f(x) =$ _____

(b) $\lim_{x \to 4^+} f(x) =$ _____

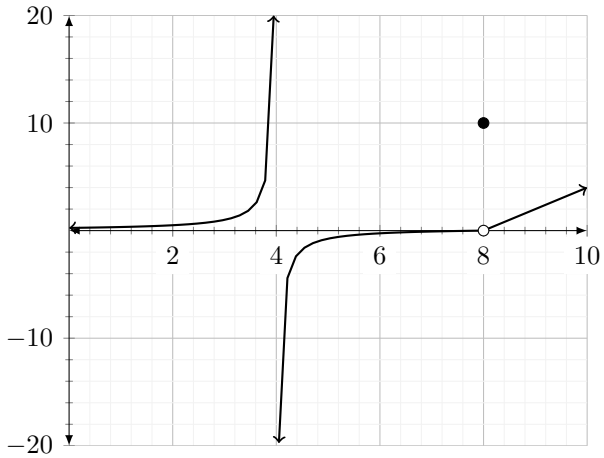
(c) $\lim_{x \to 4} f(x) =$ _____

(d) $f(4) =$ _____

(e) $\lim_{x \to 8} f(x) =$ _____

(f) $f(8) =$ _____

8. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x \to 4^-} f(x) =$ _____

(b) $\lim_{x \to 4^+} f(x) =$ _____

(c) $\lim_{x \to 4} f(x) =$ _____

(d) $f(4) =$ _____

(e) $\lim_{x \to 8} f(x) =$ _____

(f) $f(8) =$ _____

9. Find any vertical asymptotes of $f(x) = \frac{2}{x+5}$ and *justify* your answer using a limit.

10. Sketch the graph of an function that satisfies *all* of the given conditions. Compare your answer with that of your neighbor.

$\lim_{x \to 0^-} f(x) = 1$ $\lim_{x \to 0^+} f(x) = -2$ $\lim_{x \to 4^-} f(x) = 3$ $\lim_{x \to 4^+} f(x) = 0$

$f(0) = -2$ $f(4) = 1$