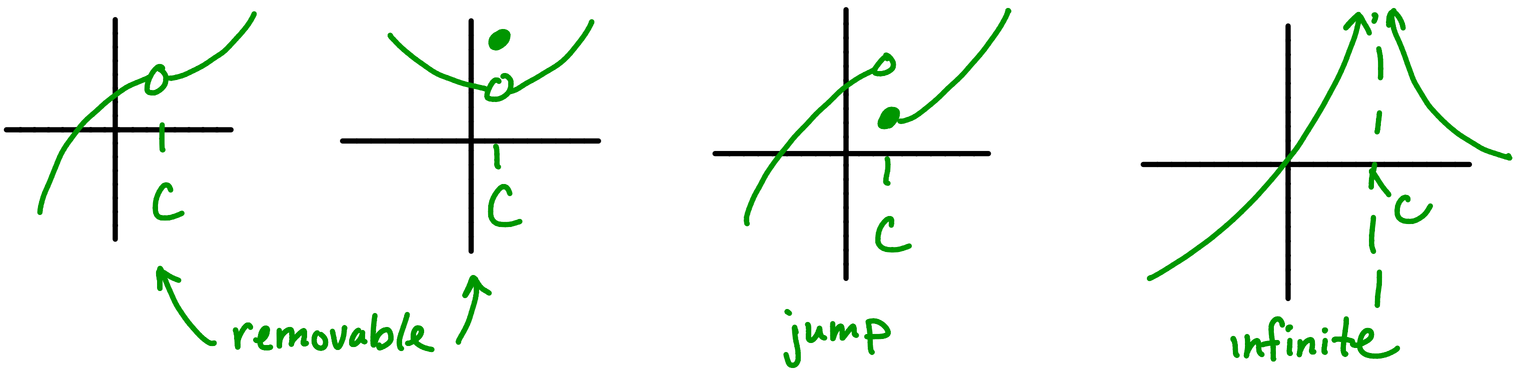
Read Section 2.4. Work the embedded problems.

1. Pictures of graph discontinuities



2. Definition of continuity at a point

short version: $f(x)$ is continuous at $x=c$ if $\lim_{x\to c} f(x) = f(c)$

check list version: $f(x)$ is continuous at $x=c$ if

☐ $\lim_{x\to c} f(x)$ exists

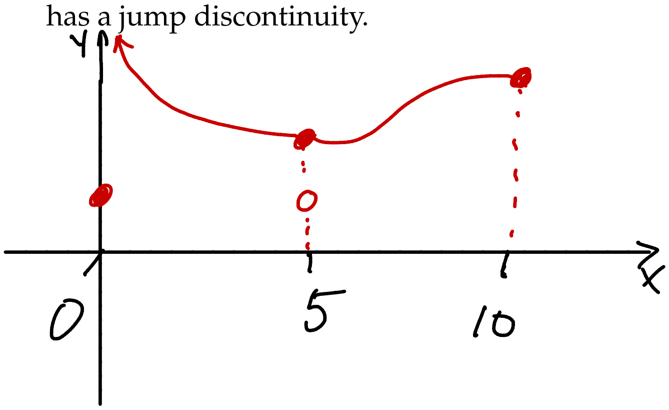
☐ $f(c)$ exists

☐ The previous $\boxed{\text{two numbers}}$ are equal.

3. Sketch the graph of a function $f(x)$ with the following properties:

   (a) the domain of $f(x)$ is the interval $[0, 10]$.

   (b) $f(x)$ is continuous except at $x = 0$ where it has in infinite discontinuity and $x = 5$ where it has a jump discontinuity.



4. Give an example of a function that is continuous everywhere on its domain.

$f(x) = x^2$, $f(x) = \sin(x)$, $f(x) = e^x$,

$f(x) = \sqrt{x}$, $f(x) = \ln(x)$

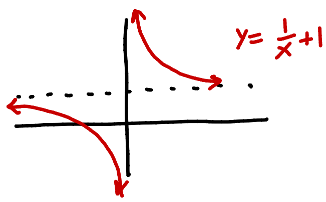
5. Determine the point(s), if any, at which each function is discontinuous. Justify your answer. Classify any discontinuity as jump, removable, infinite, or other.

   (a) $g(x) = x^{-1} + 1$ $= \frac{1}{x} + 1$ (So $x=0$ will be a problem!)

Answer: • $g(x)$ is not continuous at $x=0$.
• The discontinuity is infinite.

Justification: $\lim_{x \to 0^+} \frac{1}{x} + 1 = +\infty$, $\lim_{x \to 0^-} \frac{1}{x} + 1 = -\infty$.

Picture to confirm our work



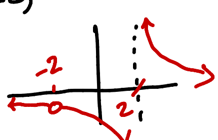
   (b) $h(x) = \frac{x+2}{x^2-4}$ $= \frac{x+2}{(x+2)(x-2)}$ (So $x=2$ and $x=-2$ will be a problem)

Answer: $h(x)$ is discontinuous at $x=2$ and $x=-2$.
At $x=2$, the discontinuity is infinite.
At $x=-2$, the discontinuity is removable.

Justification: • $\lim_{x \to -2} \frac{x+2}{(x+2)(x-2)} = \lim_{x \to -2} \frac{1}{x-2} = \frac{-1}{4}$ but $h(-2)$ is undefined.
So h has a removable discontinuity at $x=-2$

• $\lim_{x \to 2^+} \frac{x+2}{(x+2)(x-2)} = +\infty$. So h has an infinite discontinuity at $x=2$

(picture) →



6. Find the value(s) of k that makes the function continuous over the given interval.

$$f(x) = \begin{cases} e^{kx} & \text{if } 0 \le x < 4 \\ 2x+1 & \text{if } 4 \le x \le 10 \end{cases}$$

We need left- and right-limits to be equal.

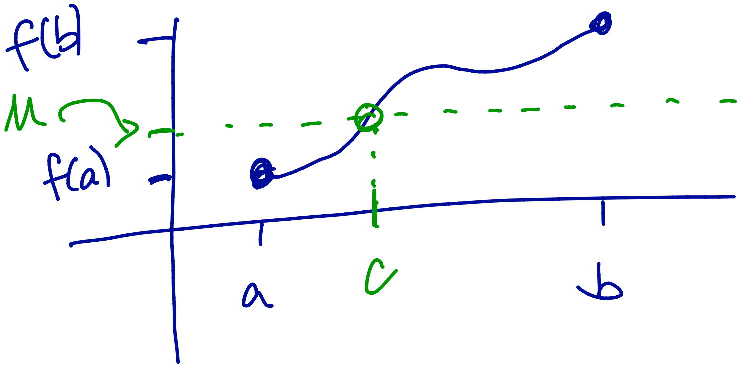
$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} 2x+1 = 2 \cdot 4 + 1 = 9$.

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} e^{kx} = e^{4k}$. So we need $e^{4k} = 9$ or $4k = \ln 9$ or $k = \frac{1}{4} \ln 9$

7. The Intermediate Value Theorem

If $f(x)$ is continuous on $[a,b]$ and M is a y-value between $f(a)$ and $f(b)$, then there is an x-value c in the open interval (a,b) so that $f(c)=M$.

[Nutshell: If $f(x)$ is continuous, it can't skip over values!]



BONUS:

8. Use the Intermediate Value Theorem to show that the equation $x^4 + x - 3 = 0$ must have a solution in the interval from $x = 1$ to $x = 2$.

Let $f(x) = x^4 + x - 3$.

Observe that $f(x)$ is continuous on $[1,2]$, because $f(x)$ is continuous everywhere

We check that $f(1) = 1+1-3 = -1 < 0$ and

$f(2) = 16+2-3 = 15 > 0$.

Since $f(1) < 0 < f(2)$ and $f(x)$ is continuous on $[1,2]$, there is some c in $(1,2)$ so that $f(c)=0$, by the IVThm. That c-value is a solution to

$x^4 + x - 3 = 0$.