Goals: To establish and justify several derivative rules and use them and to learn some new notation. Just FYI but on Wednesday we will begin with a complete and comprehensive summary of *all* the rules from this section.

← typical midterm problem!

1. Use the definition to find the derivative of $f(x) = x^2$.

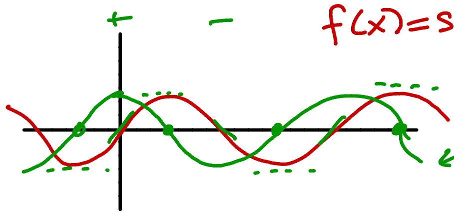
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0} 2x+h = 2x+0 = 2x.$$

Conclusion: If $f(x)=x^2$, then $f'(x) = 2x$.

$$\boxed{\text{Notation: } \frac{d}{dx}[x^2] = 2x}$$

2. Recall that at the end of class on Friday we established: If $f(x) = \sin(x)$, then $f'(x) = \cos x$

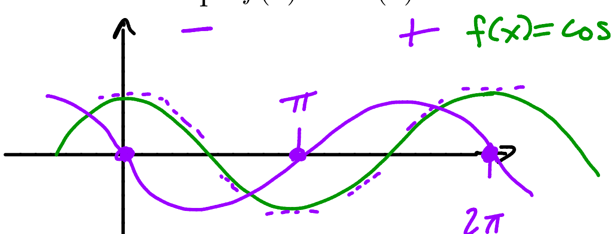


look at slopes.

← looks like cosine!

$$\boxed{\frac{d}{dx}[\sin(x)] = \cos(x)}$$

3. Graph $f(x) = \cos(x)$ and use the same strategy to guess its derivative.



← compare our guess of f' to $f(x) = \sin(x)$ above. It looks like $f'(x) = -\sin(x)$

$$\boxed{\frac{d}{dx}[\cos(x)] = -\sin(x)}$$

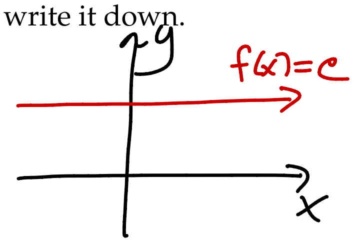
4. If $f(x) = 10$, what should $f'(x)$ be and why?

$f'(x) = 0$ b/c $f(x)$ is a horizontal line; so slope is zero.

$$\frac{d}{dx}[10] = 0$$

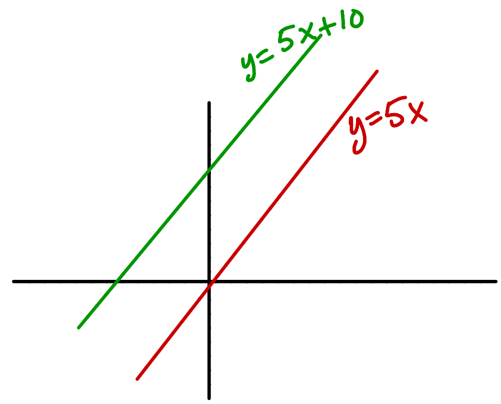
5. Make a conjecture about the derivative of constant functions and write it down.

For any constant c, $\frac{d}{dx}[c] = 0$



6. If $f(x) = x$, what should $f'(x)$ be and why?

$\frac{d}{dx}[x] = 1$ b/c its graph has slope 1.



7. What about $f(x) = 5x$? Explain.

$\frac{d}{dx}[5x] = 5$ b/c its graph has slope 5.

8. What about $f(x) = 5x + 10$? Explain.

$\frac{d}{dx}[5x+10] = 5$ b/c its graph has a slope of 5. The "+10" just shifts the graph up, but doesn't change the slope.

9. In the 3.2 notes on the definition of the derivative, we found that if $f(x) = \sqrt{x+5}$, then its derivative was: $f'(x) = \frac{1}{2\sqrt{x+5}} = \frac{1}{2}x^{-1/2}$

$\sqrt{x+5}$ " $(x+5)^{1/2}$

Use this to determine the derivative of $g(x) = \sqrt{x}$. $g'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$

$\sqrt{x}$ " $x^{1/2}$

$$\boxed{\frac{d}{dx}\left[x^{\frac{1}{2}}\right] = \frac{1}{2}x^{-1/2}}$$

10. The Power Rule

$$\frac{d}{dx}\left[x^n\right] = n\,x^{n-1}$$

See examples

$$\frac{d}{dx}\left[x^3\right] = 3x^{3-1} = 3x^2$$

$$\frac{d}{dx}\left[x^{-15}\right] = -15x^{-15-1} = -15x^{-16}$$

$$\frac{d}{dx}\left[x^{\frac{8}{3}}\right] = \frac{8}{3}x^{\frac{8}{3}-1} = \frac{8}{3}x^{\frac{5}{3}}$$

11. The Sum (and Difference) Rule

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$$

$$\frac{d}{dx}\left[f(x)-g(x)\right] = \frac{d}{dx}\left[f(x)\right] - \frac{d}{dx}\left[g(x)\right]$$

- If $H(x) = x + \sin(x)$, then $H'(x) = 1 + \cos(x)$.
- If $K(x) = x^2 - x^3$, then $K'(x) = 2x - 3x^2$.

12. The Constant Multiple Rule

$$\frac{d}{dx}[c f(x)] = c \frac{d}{dx}[f(x)]$$

If $f(x) = 10\sin(x)$, then $f'(x) = 10\cos(x)$.

If $g(x) = 7x^5$, then $g'(x) = 7 \cdot 5x^4 = 35x^4$.

13. Apply the rules to find the derivative of: (Simplify. Write w/ pos. exponents.)

(a) $f(x) = e^3$ ← a constant.

$f'(x) = 0$

(b) $f(x) = x^{-4}$

$f'(x) = -4x^{-5} = \dfrac{-4}{x^5}$

Why is the constant 4 treated differenlty than the constant 15?

(c) $H(x) = 4x^{3/2} + 15$

$H'(x) = 4\left(\frac{3}{2}x^{\frac{3}{2}-1}\right) + 0 = \frac{6}{2}x^{\frac{1}{2}}$

$$\begin{array}{r} \overset{2}{2.3} \\ \times\ 8 \\ \hline 18.4 \end{array}$$

(d) $j(x) = \dfrac{\sqrt{2}}{2} + x - 8x^{2.3}$

$j'(x) = 0 + 1 - 8(2.3)x^{2.3-1} = 1 - 18.4x^{1.3}$

14. Notation given $y = f(x)$

(first) derivative: $f'(x)$, y', $y'(x)$, $\dfrac{dy}{dx}$, $\dfrac{df}{dx}$, $\dfrac{d}{dx}(f)$

15. Higher Order Derivatives

$y = x^3 + x$

first $y' = 3x^2 + 1$

second $y'' = 6x$

third $y''' = 6$ → fourth $y^{(4)} = 0$

$f', f'', f''', f^{(4)}, \ldots$

$\dfrac{dy}{dx}, \dfrac{d^2y}{dx^2}, \dfrac{d^3y}{dx^3}, \ldots$

16. Find examples of $f(x)$ and $g(x)$ that demonstrate that the rules below are WRONG.

INCORRECT: If $H(x) = f(x)g(x)$, then $H'(x) = f'(x)g'(x)$.

Pick $f(x)=10$, $g(x)=x^2$

$H(x)=10x^2$.

So $H'(x)=20x$.

If you try to use the incorrect rule:

$f'(x)\cdot g'(x) = 0 \cdot x^2 = 0$ wrong!

INCORRECT: If $H(x) = \frac{f(x)}{g(x)}$, then $H'(x) = \frac{f'(x)}{g'(x)}$.

$H(x) = \frac{10}{x^2} = 10x^{-2}$

$H'(x) = 10(-2x^{-3})$

$= \frac{-20}{x^3}$

If you try to use the incorrect rule:

$\frac{f'}{g'} = \frac{0}{3x^2} = 0$ ← wrong!

17. Product and Quotient Rules

(a) $\frac{d}{dx}[f(x)\,g(x)] = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$H(x)=10x^2$

$H'(x) = 10\cdot 2x + 0\cdot x^2$ You would never do this normally!

$= 20x$

$K(x) = x\sin(x)$

$K'(x) = x\cdot\cos(x) + 1\cdot\sin(x)$

$= x\cos(x) + \sin(x)$

(b) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{[g(x)]\cdot f'(x) - [f(x)]\,g'(x)}{[g(x)]^2}$

$K(x) = \frac{\cos(x)}{x^2+5}$; $K'(x) = \frac{(x^2+5)(-\sin(x)) - (\cos(x))(2x+0)}{(x^2+5)^2}$

$= \frac{-((x^2+5)\sin(x) + 2x\cos(x))}{(x^2+5)^2}$