SECTION 3-4: DERIVATIVES AS RATES OF CHANGE

Read Section 3.4. Work the embedded problems.

1. The cost in dollars of producing x widgets is given by $C(x) = 6000 + 10x + 0.01x^2$. $C'(x) = 10 + 0.02x$

C′ is called marginal cost. "marginal" means derivative.

(a) Find $C(100)$ and $C'(100)$. Include units with your answer.

$C(100) = 6000 + 1000 + (0.01)(10000)$
$= 8000$ dollars.

$C'(100) = 10 + (0.02)(100) = 10 + 2$
$= 12$ \$/widget

(b) Explain what the numbers in part (a) mean in the context of the problem.

To produce 100 widgets will cost \$8000. The rate of change of cost per widget is \$12/widget when producing 100 widgets.

(c) If the marginal profit of the widget-making company was negative, what should the company conclude about production?

$P'(x) < 0$ If marginal profit is negative, it would indicate the profit was decreasing with increasing production. That is, producing more will decrease profit.

2. Suppose $p(t)$ gives the number of bacteria after t hours in some lab experiment.

(a) Interpret $p(10) = 1000$ and $p'(10) = 20$.

At 10 hours, there are 1000 bacteria and the population at this time is increasing at a rate of 20 bacteria per hour.

(b) Estimate the number of bacteria when $t = 11$.

$1000 + 20 = 1020$ bacteria

3. Suppose $s(t)$ gives the position of an object where s is measured in feet and t is measured in seconds.

(a) Determine the units of $s'(t)$ and $s''(t)$ and interpret them in the context of the problem.

s' is velocity measured in ft/s. s'' is acceleration measured in ft/sec^2

(b) If $s'(5) = 20$ and $s''(5) = 2$, estimate $s'(6)$. Is the object speeding up or slowing down?

$s'(6) \approx s'(5) + s''(5) = 22$. Speeding up (from $v=20$ to $v=22$)

(c) If $s'(5) = 20$ and $s''(5) = -2$, estimate $s'(6)$. Is the object speeding up or slowing down?

$s'(6) \approx s'(5) + s''(5) = 20 - 2 = 18$. Slowing down (from $v=20$ to $v=18$).

4. A potato is launched vertically upward from a platform 20 feet off the ground. The distance in feet that the potato travels from the ground after t seconds is given by $s(t) = -16t^2 + 64t + 20$.

(a) Find the initial velocity of the potato.

$v(t) = s'(t) = -32t + 64$

initial velocity $= v(0) = 64$ ft/s

(b) Find the velocity and the acceleration of the potato when $t = 1$.

$v(1) = -32(1) + 64 = 32$ ft/s

$a(t) = -32$, $a(3) = -32$ ft/sec^2

(c) Is the potato speeding up or slowing down? Why?

Slowing down.
Because $v > 0$ and $a < 0$.

(d) What is the velocity of the potato when it reaches its maximum height and why?

at max height, $v = 0$.
This is the moment velocity changes from positive to negative. So it must be zero.

(e) What is the maximum height of the potato?

(find time t at max height: $v(t) = 0 = -32t + 64$. So $t = 2$s.

(plug time into position equation:) $s(2) = -16(2^2) + 64(2) + 20 = 84$ ft

(f) Assume the potato lands on the ground (not the platform). How long is the potato in the air?

(find time when potato lands:) $s(t) = 0 = -16t^2 + 64t + 20 = 0$

(use quad. formula:) $t = \dfrac{-64 \pm \sqrt{64^2 - 4(-16)(20)}}{2(-16)} \approx \frac{1}{2}(4 + \sqrt{21}) \approx 4.2913$ sec

(g) What is the velocity of the potato when it hits the ground?

$v(4.2913) = -32(4.2913) + 64 = -73.32$ ft/s

(h) You should have observed in part (b) that the acceleration is constant. What does this number represent?

acceleration due to gravity on earth.