SECTION 3-5: DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

Read Section 3.5. Work the embedded problems.

1. (Quick Review)

   (a) What's wrong with the solution below? The student's solution is in large font. They were instructed to put a box around their final answer.

   Problem: Find the derivative of $f(x) = 5\sin(x) - \frac{x}{3} + \frac{\pi^2}{3}$

   $= 5\sin(x) - \frac{1}{3}x + 0 = \boxed{-5\cos(x) - \frac{1}{3}}$

   None of these equalities is correct.
   It looks like the student doesn't understand the difference between $f(x)$ and $f'(x)$.

   (b) If $f(x) = \cos(x)$, then $f'(x) = -sin(x)$.

2. Write all the Pythagorean Trigonometric Identities

   $\sin^2(x) + \cos^2(x) = 1$ (divide by $\cos^2 x$)

   $\tan^2(x) + 1 = \sec^2(x)$

   (divide by $\sin^2(x)$)

   $1 + \cot^2(x) = \csc^2(x)$

3. Rewrite each trigonometric function below in terms of sines and cosines, then use known derivative rules (product, quotient, sine, cosine, etc.) to find their derivatives.

   (a) $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$

   $f'(x) = \frac{\cos(x)(\cos(x)) - (\sin(x))(-\sin(x))}{\cos^2(x)} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2(x)} = \boxed{\sec^2 x}$

   (b) $f(x) = \cot(x) = \frac{\cos(x)}{\sin(x)}$

   $f'(x) = \frac{\sin(x)(-\sin(x)) - \cos(x)(\cos(x))}{\sin^2(x)} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x}$

   $= \boxed{-\csc^2 x}$

   (c) $f(x) = \sec(x) = \frac{1}{\cos x}$

   $f'(x) = \frac{(\cos(x))(0) - 1(-\sin(x))}{\cos^2 x} = \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)} = \boxed{(\tan(x))(\sec(x))}$

(d) $f(x) = \csc(x) = \frac{1}{\sin(x)}$

$$f'(x) = \frac{\sin(x)\cdot 0 - 1(\cos(x))}{\sin^2(x)} = \frac{-\cos(x)}{\sin^2(x)} = -\frac{\cos(x)}{\sin(x)}\cdot\frac{1}{\sin(x)} = -\cot(x)\csc(x)$$

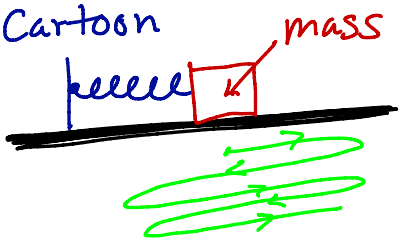
4. SUMMARY RULES:

Do you see the pattern?

$$\frac{d}{dx}[\tan(x)] = \sec^2(x) \qquad \frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$$

$$\frac{d}{dx}[\cot(x)] = -\csc^2(x) \qquad \frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$$

5. A mass on a spring vibrates horizontally on a smooth level surface. Its equation of motion is $x(t) = 8\sin(t)$, where t is in seconds and x is in centimeters.

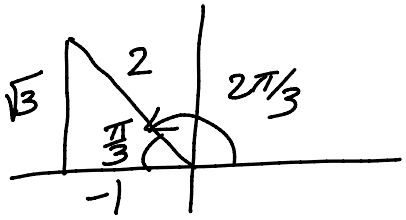


(a) Find the velocity and acceleration at time t.

$v(t) = x'(t) = 8\cos(t)$ ← cm/sec

$a(t) = x''(t) = -8\sin(t)$ ← cm/sec^2

Use this →



(b) Find the position, velocity, and acceleration of the mass at time $t = 2\pi/3$. In what direction is it moving at this time? Is it speeding up or slowing down?

$x(2\pi/3) = 8\sin(2\pi/3) = 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$ cm

$v(t) = 8\cos(2\pi/3) = 8\left(\frac{-1}{2}\right) = -4$ cm/sec

$a(t) = -8\sin(2\pi/3) = -8\left(\frac{\sqrt{3}}{2}\right) = -4\sqrt{3}$ cm/sec^2

Direction?
- to the left b/c $v < 0$

Speeding up/slowing?
- speeding up b/c v and a are both negative.

(d) $f(x) = \csc(x)$

4. SUMMARY RULES:

5. A mass on a spring vibrates horizontally on a smooth level surface. Its equation of motion is $x(t) = 8\sin(t)$, where t is in seconds and x is in centimeters.

   (a) Find the velocity and acceleration at time t.

   (b) Find the position, velocity, and acceleration of the mass at time $t = 2\pi/3$. In what direction is it moving at this time? Is it speeding up or slowing down?