1. Recall Two Versions of the Chain Rule

$\boxed{A}$ $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\cdot g'(x)$

$\boxed{B}$ $y=f(u)$, $u=g(x)$ $\qquad \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

2. Understanding what the "formulas" in the book are trying to communicate:

(Thm 3.10)

$$\frac{d}{dx}\left[\tan(g(x))\right] = \sec^2(g(x))\cdot g'(x)$$

$$\frac{d}{dx}\left[\tan(u)\right] = \sec^2(u)\cdot\frac{du}{dx}$$

The goal is to help you read math textbooks. Can you produce the secant "rule"?

Chain Rule for Composition of Three Functions

If $K(x) = h(f(g(x)))$

then $K'(x) = h'(f(g(x)))\cdot\frac{d}{dx}\left[f(g(x))\right]$

$= h'(f(g(x)))\cdot f'(g(x))\cdot g'(x)$

3. Find the derivatives for each function below:

(a) $f(\theta) = 4\tan(\theta/\pi)$. $= 4\cdot\tan(\frac{1}{\pi}\cdot\theta)$

$$f'(\theta) = 4\cdot\sec^2\left(\frac{1}{\pi}\theta\right)\cdot\frac{1}{\pi} = \frac{4}{\pi}\sec^2\left(\frac{1}{\pi}\theta\right)$$

(b) $g(t) = \sqrt[5]{\sin(7t)}$ $= (\sin(7t))^{\frac{1}{5}}$

(follow the "rule"): $g'(t) = \frac{1}{5}(\sin(7t))^{-4/5}(\cos(7t))\cdot 7 = \frac{7\cos(7t)}{5(\sin(7t))^{4/5}}$

(just think about it): $g'(t) = \frac{1}{5}(\sin(7t))^{-4/5}\cdot\frac{d}{dt}\left[\sin(7t)\right] = \frac{1}{5}(\sin(7t))^{-4/5}\cdot\cos(7t)\cdot\frac{d}{dt}\left[7t\right]$

$= \frac{1}{5}(\sin(7t))^{-4/5}\cos(7t)(7)$

(c) $h(x) = \sin(x^2 - \frac{1}{x^2+x})$

$= \sin\left(x^2 - (x^2+x)^{-1}\right)$

$$h'(x) = \cos\left(x^2 - (x^2+x)^{-1}\right)\cdot\frac{d}{dx}\left[x^2 - (x^2+x)^{-1}\right]$$

$$= \cos(x^2-(x^2+x)^{-1})\cdot\left(2x - (-1)(x^2+x)^{-2}\cdot\frac{d}{dx}(x^2+x)\right)$$

$$= \cos\left(x^2-(x^2+x)^{-1}\right)\left(2x + (x^2+x)^{-2}(2x+1)\right)$$

* See $\boxed{\text{Note}}$ on last page!

4. (Some additional independent practice) Find the derivatives.

(a) $f(x) = (\sec(3x) + \csc(2x))^5$

$$f'(x) = 5\left(\sec(3x) + \csc(2x)\right)^4 \cdot \left(3\sec(3x)\tan(3x) - 2\csc(2x)\cot(2x)\right)$$

(b) $g(x) = \frac{\cot(x^2+1)}{x^3+1}$

$$g'(x) = \frac{(x^3+1)\left(-\csc^2(x^2+1)(2x)\right) - \cot(x^2+1)(3x^2)}{(x^3+1)^2}$$

(c) $h(x) = (2x-1)^3(2x+1)^5$

$$h'(x) = \underbrace{3(2x-1)^2(2)}_{f'}\underbrace{(2x+1)^5}_{g} + \underbrace{(2x+1)^3}_{f} \cdot \underbrace{5\cdot(2x+1)^4(2)}_{g'}$$

5. Find all x-values where the tangent to $f(x) = (x^2-4)^3$ is horizontal. $f' = 0$.

$$f'(x) = 3(x^2-4)^2(2x) = 6x(x^2-4) = 0$$

So $x = 0$ or $x^2 - 4 = 0$.

$x^2 - 4 = 0$ when $x^2 = 4$ or $x = \pm 2$

Answer: $f(x)$ has a horizontal tangent when $x = -2, 0, 2$.

6. Use the table below to evaluate the derivatives of the given functions at the indicated value.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	2	-1	0	1
0	1	2	3	4
1	-1	-2	-3	-4
2	0	4	-1	2

(a) $h(x) = f(g(x))$ at $a = 2$.

$h'(x) = f'(g(x))\cdot g'(x)$; $h'(2) = f'(g(2))\cdot g'(2) = f'(-1)\cdot 2 = -2$

(b) $k(x) = f(x)g(x^2)$ at $a = 1$

$k'(x) = f'(x)\cdot g(x^2) + f(x)\cdot g'(x^2)(2x)$

$k'(1) = f'(1)\cdot g(1) + f(1)g'(1)(2\cdot 1) = (-2)(-3) + (-1)(-4)(2)$

$= 6 + 8 = 14$

Note

(c) $h(x) = \sin(x^2 - \frac{1}{x^2+x})$

$$= \sin\left(x^2 - (x^2+x)^{-1}\right)$$

$$h'(x) = \cos\left(x^2 - (x^2+x)^{-1}\right) \cdot \frac{d}{dx}\left[x^2 - (x^2+x)^{-1}\right]$$

$$= \cos\left(x^2-(x^2+x)^{-1}\right) \cdot \left(2x - (-1)(x^2+x)^{-2} \cdot \frac{d}{dx}(x^2+x)\right)$$

$$= \cos\left(x^2-(x^2+x)^{-1}\right)\left(2x + (x^2+x)^{-2}(2x+1)\right)$$

This was our answer. It is correct.

Do you see the difference between the correct answer (above) and the incorrect answer below?

$$h'(x) = \cos\left(x^2-(x^2+x)^{-1}\right)\left(2x - (-1)(x^2+x)^{-2}\right)(2x+1)$$

- Look at the parantheses.

In the two expressions (correct + incorrect) compare what the term "2x+1" is multiplied by.