1. Quick Review

$\frac{d}{dx}[a^x] = (\ln a) a^x$

$\frac{d}{dx}[e^x] = e^x$

$\frac{d}{dx}[\ln x] = \frac{1}{x}$

$\boxed{y = \ln x} \overset{\text{or}}{\equiv} \boxed{x = e^y}$

$1 = e^y \frac{dy}{dx}$

or $\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{e^{\ln x}} = \frac{1}{x}$

$\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$

2. Find the derivatives for the functions below.

(a) $f(x) = x\ln(x)$

$f'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x}$

$= \ln(x) + 1$

(b) $f(x) = \log_2(x^2 + \sin(x))$

$f'(x) = \frac{1}{(\ln 2)(x^2 + \sin(x))} \cdot (2x + \cos(x))$

$= \frac{2x + \cos(x)}{(\ln 2)(x^2 + \sin(x))}$

(c) $f(x) = \frac{1}{\sqrt{1+x+\ln(1+3x)}}$

$= (1 + x + \ln(1+3x))^{-1/2}$

$f'(x) = -\frac{1}{2}(1 + x + \ln(1+3x))^{-3/2}\left(1 + \frac{3}{1+3x}\right)$

(d) $f(x) = \ln\left(\frac{x^4}{(x+1)^2}\right) = 4\ln x - 2\ln(x+1)$

$f'(x) = 4 \cdot \frac{1}{x} - 2\left(\frac{1}{x+1}\right)$

$= \frac{4}{x} - \frac{2}{x+1}$

3. Logarithmic Differentiation: A Strategy for Finding Even More Derivatives

(a) $y = x^x$

clever trick

$\ln(y) = \ln(x^x) = x\ln x$

take derivative implictly.

$\frac{1}{y}\frac{dy}{dx} = 1\cdot\ln x + x\cdot\frac{1}{x}$

$\frac{dy}{dx} = y(\ln x + 1)$

$= (x^x)(\ln x + 1)$

(b) $y = (x^2+1)^{\sin(x)}$

$\ln y = \sin(x)\ln(x^2+1)$

$\frac{1}{y}\frac{dy}{dx} = \cos(x)(\ln(x^2+1)) + \sin(x)\left(\frac{2x}{x^2+1}\right)$

$\frac{dy}{dx} = (y)\left[\cos(x)(\ln(x^2+1)) + \frac{2x\sin(x)}{x^2+1}\right]$

$= (x^2+1)^{\sin x}\left[\cos(x)(\ln(x^2+1)) + \frac{2x\sin(x)}{x^2+1}\right]$

(c) $y = \frac{xe^x}{\sqrt{1+7x}}$

$\ln y = \ln\left(\frac{xe^x}{\sqrt{1+7x}}\right) = \ln x + \ln(e^x) - \ln\left(\sqrt{1+7x}\right) = \ln x + x - \frac{1}{2}\ln(1+7x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + 1 - \frac{1}{2}\left(\frac{7}{1+7x}\right)$

$\frac{dy}{dx} = y\left(\frac{1}{x} + 1 - \frac{7}{2(1+7x)}\right) = \left(\frac{xe^x}{\sqrt{1+7x}}\right)\left(\frac{1}{x} + 1 - \frac{7}{2(1+7x)}\right)$