1. Quick Review Differentiation:

   (a) Find dy/dx for $x^2 - y^3 = x\sin(y)$.

   $2x - 3y^2\frac{dy}{dx} = 1\cdot\sin(y) + x\cos(x)\frac{dy}{dx}$

   $2x - \sin(y) = \frac{dy}{dx}(3y^2 + x\cos(x))$

   $\frac{dy}{dx} = \frac{2x-\sin(y)}{3y^2+x\cos(x)}$

   (b) Find y' for $y = x(\sin(x))^{-1}$ $= x\csc x$

   $y' = 1\cdot(\sin(x))^{-1} + x(-1)(\sin(x))^{-2}(\cos(x))$

   $= \frac{1}{\sin(x)} - \frac{x\cos(x)}{(\sin(x))^2}$

   $y' = 1\cdot\csc x + x(-\csc x\cot x)$

   $= \csc x - x\csc x\cot(x)$

   (c) Find y' for $y = x\sin^{-1}(x)$

   $y' = 1\cdot\sin^{-1}(x) + x\cdot\frac{1}{\sqrt{1-x^2}} = \sin^{-1}(x) + \frac{x}{\sqrt{1-x^2}}$

2. Let $f(x) = e^x$. **Estimate** $f'(x)$ (a.k.a. the slope of the tangent line) using the slope of a secant line for each of the values below. (Use a calculator!)

   Pattern?

   Note.

   (a) $f'(0)$ $\approx \frac{e^{0.001} - e^0}{0.001 - 0} = 1.0005 \approx f(0) = 1$

   * $f(0)=1$

   (b) $f'(1) \approx \frac{e^{1.001} - e^1}{1.001 - 1} = 2.71964 \approx f(1) = e^1 = e = 2.71828$

   (c) $f'(2) \approx \frac{e^{2.001} - e^2}{2.001 - 2} = 7.39275 \approx f(2) = e^2 = 7.38905$

   (d) $f'(-1) \approx \frac{e^{-1.001} - e^{-1}}{-1.001 - (-1)} = 0.36769 \approx f(-1) = e^{-1} = 0.36787$

   $f(a) = f'(a)$?!!

   y-values = derivative values?!

3. Derivative Rules for Exponential Functions

   $\frac{d}{dx}[e^x] = e^x$

   Again, y-values equal derivative values

   $\frac{d}{dx}[a^x] = (\ln a)a^x$

   Note: $a^x = e^{(\ln a)x}$.

   Do you see the relationship?

4. Examples:

(a) $y = \overset{f}{x^4}\overset{g}{e^x}$

$y' = 4xe^x + x^4 e^x$

$f' \cdot g + f \cdot g'$

(b) $y = e^{x^2} = e^{(x^2)}$ ← chain rule!

$y' = (e^{x^2})(2x) = 2x e^{x^2}$

(c) $y = 5^{-x} = 5^{(-x)}$ ← chain rule!

$y' = ((\ln 5) 5^{-x})(-1)$

$= (-\ln 5) 5^{-x}$

Alternate:

$y = \left(\frac{1}{5}\right)^x$

$y' = \ln\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)^x$

(d) $f(x) = x^5 + 5^x$

$f'(x) = \underbrace{5x^4}_{\text{power rule}} + \underbrace{(\ln 5) 5^x}_{\text{exponential rule}}$ ← Do you see why different rules are used?

5. Let $P(t) = P_0 e^{kt}$. Write $P'(t)$ in terms of $P(t)$.

$P'(t) = (P_0)(e^{kt})(k)$

$= P_0 k e^{kt}$. → Rewrite in terms of $P(t)$

$P(t) = P_0 e^{kt}$

So $P'(t) = k P(t)$ ||

In words: $P'(t)$ is proportional to $P(t)$.

6. A population of bacteria has an initial population of 200 bacteria. The population is growing at a rate of 4 % per hour. → $P' = 0.04 P$ or $k = 0.04$

(a) Write an exponential function $P(t)$ that relates the total population as a function of t where the units of t should be hours and the units of P should be number of bacterial.

$P(t) = 200 e^{0.04t}$

Check: $P(0) = 200 e^0 = 200$

$P(1) = 200 e^{0.04} = 208.162$

$\frac{208-200}{200} = \frac{8}{200} = \frac{4}{100} = 4\%$

(b) Find and interpret $P'(1)$.

$P'(t) = (200)(0.04) e^{0.04t} = 8 e^{0.04t}$

$P'(1) = 8 e^{0.04} \approx 8.3264.$

// At 1 hour, the population is increasing at a rate of 8 bacteria per hour.

(c) Find and interpret ~~$P'(10)$~~. $P'(100)$

$P'(100) = 8 e^4 = 436.7$

At 10 hours, the population is increasing at a rate of 437 bacteria per hour.