SECTION 4.6: LIMITS AT INFINITY AND ASYMPTOTES (DAY 2)

1. Three Principles (a is a constant)

   - If a is a constant, then $\lim\limits_{x\to\pm\infty} ax =$
   - $\lim\limits_{x\to\pm\infty} \dfrac{1}{x} =$
   - If $\lim\limits_{x\to\pm\infty} f(x) = a$ and $\lim\limits_{x\to\pm\infty} g(x) = \pm\infty$, then $\lim\limits_{x\to\pm\infty} \dfrac{f(x)}{g(x)} =$

2. Use the Principles above to evaluate the limits below.

   (a) $\lim\limits_{x\to\infty} \dfrac{-x}{3x-5x^2}$

   (b) $\lim\limits_{x\to\infty} \dfrac{2x^2-x}{3x-5x^2}$

   (c) $\lim\limits_{x\to\infty} \dfrac{2x^3-x}{3x-5x^2}$

   (d) $\lim\limits_{x\to\infty} \dfrac{3x+\sin(x)}{x}$

   (e) $\lim\limits_{x\to-\infty} \dfrac{2x+1}{\sqrt{x^2+1}}$

   (f) $\lim\limits_{x\to\infty} \dfrac{2e^x+1}{1-3e^x}$

3. Limits at Infinity and Horizontal Asymptotes: If $\lim_{x\to\infty} f(x) = L$, then

4. Find all asymptotes of $f(x) = \frac{x}{3-x}$ and *justify* your answers.

5. Given $f(x) = \frac{2}{x^2+1}$, $f'(x) = \frac{2x}{(x^2+1)^2}$, $f''(x) = \frac{-2(3x^2-1)}{(x^2+1)^3}$. Identify important features of $f(x)$ like: asymptotes, local extrema, inflection points, and make a rough sketch.