Section 5.4: The Net Change Theorem

1. Quick Review: Evaluate the following.

   (a) $\int \left(\frac{x}{3} - \sin(x)\right)\, dx$

   $= \frac{x^2}{6} + \cos(x) + C$

   (b) $\int_0^5 (3 - e^x)\, dx$

   $= 3x - e^x \Big]_0^5$

   $= (15 - e^5) - (0 - e^0)$

   $= 15 - e^5 + 1 = \underline{16 - e^5}$

   (c) $\frac{d}{dx}\left(\int_1^{x^2} (\ln(t))\, dt\right)$

   $= \ln(x^2)(2x)$

   $= 4x\ln(x).$

2. Assume $P'(t)$ gives the rate of change in a population of ants over time where time t is measured in days and $P'(t)$ is measured in hundreds of ants per day. Use the table below to answer the questions.

   1 2 3

t	0	7	14	21	28	35
P′(t)	0	1.9	2.4	2.7	3.0	3.2

   (a) Interpret $P'(14) = 2.4$.

   At day 14, the colony is adding 240 ants each day.

   (b) Estimate how much the ant population increased in the first three weeks. Include units with your answer.

   $7(1.9) + 7(2.4) + 7(2.7) = 49$. Estimate 4,900 ants

   (Many different answers here. This is probably an over-estimate)

   (c) What would $\int_0^{21} P'(t)\, dt$ represent? (~~There are many ways to answer this question. Think of as many as you can.~~ Include units ~~this that is appropriate~~ if appropriate)

   The number of ants added to the colony in the first 3 weeks. The units would be hundreds of ants.

   (d) What would $P(t)$ represent? What is $P(14)$?

   P(t) would be the # of ants in the colony.
   We don't know P(14) because we don't know how many ants the colony started with.

3. The Net Change Theorem:

   The integral of a rate of change is the net change:

   $$\int_a^b F'(t)\,dt = F(b) - F(a)$$

4. Let $w'(t)$ be the rate of growth of a child in pounds per year.

   (a) What does $\int_5^{10} w'(t)\,dt$ represent? (Write a complete sentence a regular person could understand.)

   It represents how much weight the child gained in pounds from 5 years of age to 10 years of age.

   (b) Explain what $w(10)$ represents.

   w(10) would represent the child's weight at 10 years of age.

5. Snow is falling on my garden at a rate of $m'(t) = 6t$ kilograms per hour for $0 \le t \le 2$ where t is measured in hours.

   (a) Find and interpret $m'(1)$.

   $m'(1) = 6 \cdot 1 = 6$ kg/hr. It is the rate at which snow is falling at 1 hour.

   (b) Find an interpret $\int_0^2 m'(t)\,dt$

   $\int_0^2 6t\,dt = 3t^2\Big|_0^2 = 12$ Kg

   It is the mass of snow added to my garden during the two hours.

   (c) In this context, what would $m(0) = 13$ represent?

   The mass of snow on my garden when the model started (when t=0.)

   (d) Find and interpret $m(2)$.

   $m(2) = m(0) + \int_0^2 m'(t)dt = 13 + 12 = 25$ Kg

6. The height of water in a cylindrical tank is modeled by $h'(t) = 3\sin(t)$ where h' is measured in meters per hour and t is measured in hours. It is a fact that

$$\int_0^{\pi} h'(t)dt = 6 \text{ and } \int_{\pi}^{2\pi} h'(t)dt = -6.$$

(You should check this on your own.)
Use the information to find $\int_0^{2\pi} h'(t)dt$. Can you explain what is happening in this tank? Do you think the tank is running out of water?

$\int_0^{2\pi} h'(t)dt = \int_0^{\pi} h'(t)dt + \int_{\pi}^{2\pi} h'(t)dt = 6 - 6 = 0$

The height of the water is sometimes increasing & sometimes decreasing. We don't know the height of the water. We only know that the height is fluctuating.