1. Definitions of the Derivative

Version 1

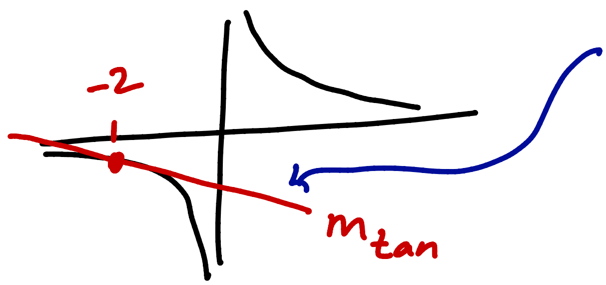
$$f'(a) = m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Version 2

$$f'(a) = m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

2. In the problems below, let $f(x) = \frac{1}{x}$.

(a) Using a *rough* sketch of $f(x)$ make a rough estimate of the slope of the tangent to $f(x)$ when $x = -2$.



looks like slope is negative + not too steep

(b) Use version 1 of the definition to find m_{tan}

when $a = -2$

$$f'(-2) = m_{tan} = \lim_{x \to -2} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{+2}}{x+2} = \lim_{x \to -2} \frac{\left(\frac{2+x}{2x}\right)}{x+2}$$

$$= \lim_{x \to -2} \left(\frac{1}{x+2}\right)\left(\frac{x+2}{2x}\right) = \lim_{x \to -2} \frac{1}{2x} = \frac{1}{-4} = -\frac{1}{4}$$

(c) Using version 1 of the definition to find m_{tan}

$$f'(-2) = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h \to 0} \frac{\frac{1}{-2+h} + \frac{1}{2}}{h}$$

$$= \lim_{h \to 0} \left(\frac{1}{h}\right)\left(\frac{2 + (-2+h)}{(-2+h)(2)}\right) = \lim_{h \to 0} \frac{h}{2(h-2)h} = \lim_{h \to 0} \frac{1}{2(h-2)} = \frac{-1}{4}$$

(d) Write the equation of the line tangent to $f(x)$ when $x = -2$. Plausible?

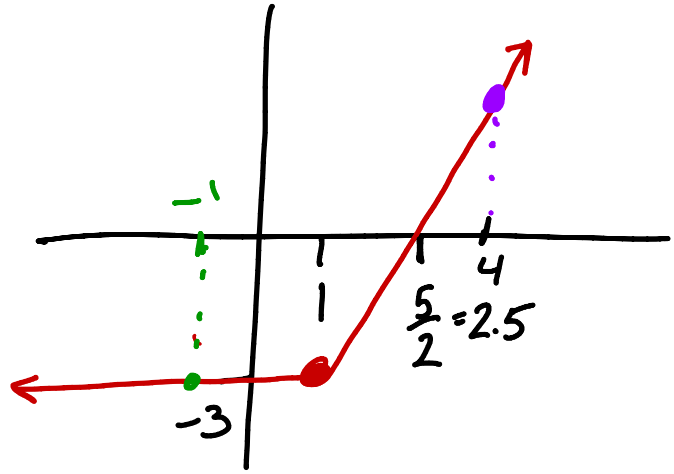
point $(-2, -\frac{1}{2})$

$m = -\frac{1}{4}$

line: $y + \frac{1}{2} = -\frac{1}{4}(x+2)$ or $y = -\frac{1}{4}x - 1$ (Looks roughly correct)

3. Graph the function $G(t) = \begin{cases} -3 & x \leq 1 \\ 2x-5 & 1 < x \end{cases}$.

(a) Use the graph to determine $G'(-1)$ and $G'(4)$



- $G'(-1)=0$
- $G'(4)=2$

(b) Explain – using the definition – why $G'(1)$ fails to exist.

The definition of the derivative involves a TWO-sided limit. On the left side of $x=1$, all secant lines have a slope of zero, but on the right side, all secant lines have a slope of 2.

OR: $\lim_{x\to 1^-} \frac{G(x)-G(1)}{x-1}=0$, but $\lim_{x\to 1^+} \frac{G(x)-G(1)}{x-1}=2$. So $\lim_{x\to 1} \frac{G(x)-G(1)}{x-1}=\text{DNE}$.

4. A rock is dropped from a height of 100 feet. Its height above ground at time t seconds later is given by $s(t) = -16t^2 + 100$.

(a) Find and interpret $s(0)$ and $s(1)$.

$s(0)=-16\cdot 0^2+100=100$ feet.
When time starts, the rock is 100 feet above the ground (like the problem says...)

$s(1)=-16(1)^2+100=84$ feet.
One second later, the rock is only 84 feet above the ground (i.e. it has fallen, which is expected.)

(b) Given $s'(1) = -32$, determine the units of $s'(1)$ and interpret it in the context of the problem.

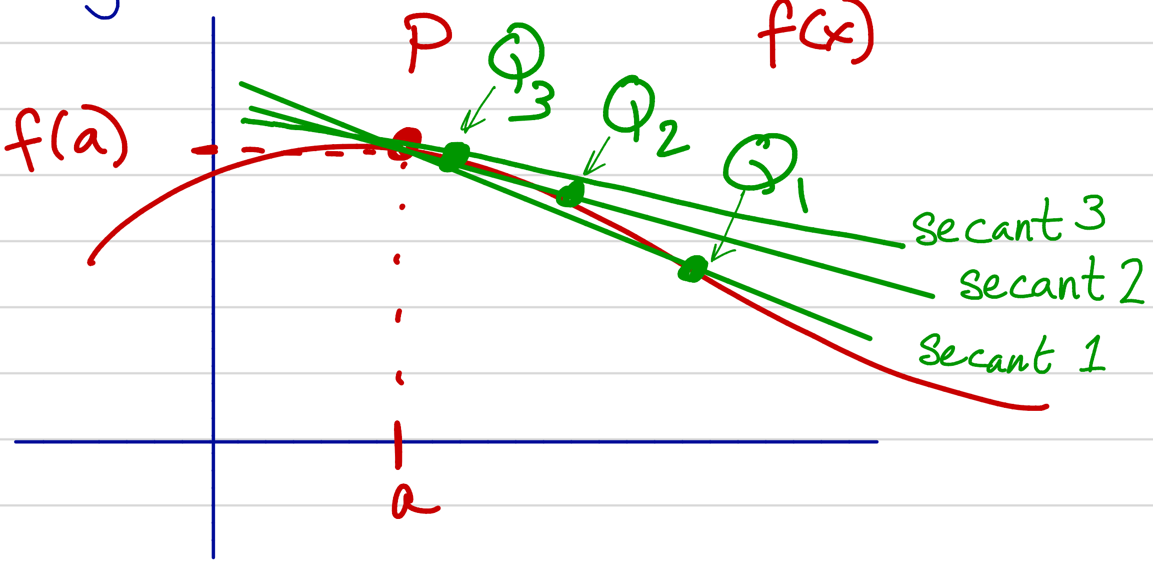
Recall that $s'(1)=\lim_{t\to 1} \frac{s(t)-s(1)}{t-1} = \frac{\Delta s}{\Delta t}$; ← velocity!

so (units of s') $= \frac{\text{units of } s}{\text{units of } t} = \frac{ft}{sec}$ ← velocity!! $\frac{\text{change in position}}{\text{change in time}}$.

When 1 second has passed, the velocity of the rock is -32 ft/s

Notes

Return to §2.1 where we approximated the slope of the tangent to f(x) at point P using secant lines



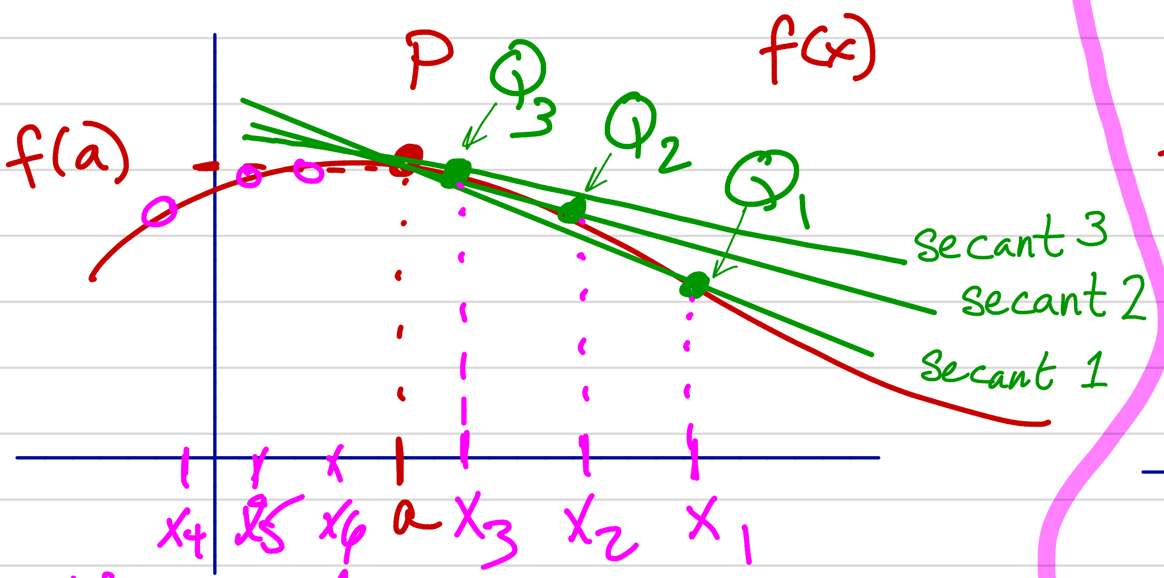
We now can see this as a limit.

As the Q's get close to the P's, m_{sec} gets close to m_{tan}.

OR

$$\boxed{\lim_{Q \to P} m_{sec} = m_{tan}}$$

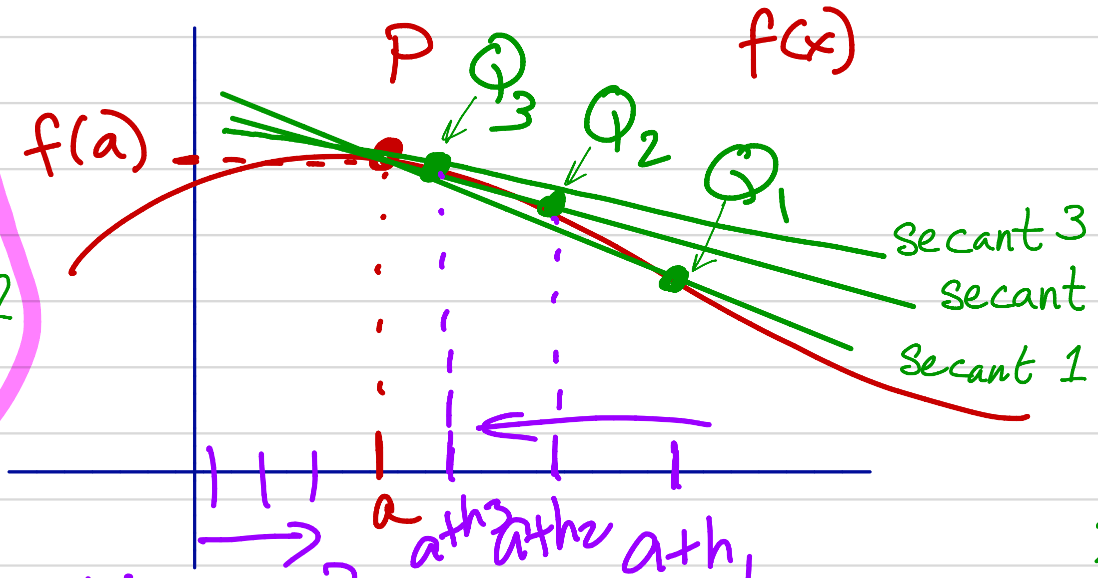
How do we make this precise?



Version 1

$$\lim_{x \to a} \left(\frac{f(x) - f(a)}{x - a} \right) = m_{tan}$$

↑ slope secant $\frac{\Delta y}{\Delta x}$



Version 2

$$\lim_{h \to 0} \left(\frac{f(a+h) - f(a)}{\underbrace{(a+h) - (\quad)}_{\text{or } a+h-a = h}} \right) = m_{tan}$$