SECTION 3-5: DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

1. Write all the Pythagorean Trigonometric Identities

$\sin^2 x + \cos^2 x = 1$

$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \Rightarrow \tan^2 x + 1 = \sec^2 x$

$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \Rightarrow 1 + \cot^2 x = \csc^2 x$

2. Rewrite each trigonometric function below in terms of sines and cosines, then use known derivative rules (product, quotient, sine, cosine, etc.) to find their derivatives.

(a) $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$

$f'(x) = \frac{\cos(x)(\cos(x)) - (\sin(x))(-\sin(x))}{\cos^2(x)} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2(x)} = \boxed{\sec^2 x}$

(b) $f(x) = \cot(x) = \frac{\cos(x)}{\sin(x)}$

$f'(x) = \frac{\sin(x)(-\sin(x)) - \cos(x)(\cos(x))}{\sin^2(x)} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x}$

$= \boxed{-\csc^2 x}$

(c) $f(x) = \sec(x) = \frac{1}{\cos x}$

$f'(x) = \frac{(\cos(x))(0) - 1(-\sin(x))}{\cos^2 x} = \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)} = \boxed{(\tan(x))(\sec(x))}$

(d) $f(x) = \csc(x) = \frac{1}{\sin(x)}$

$$f'(x) = \frac{\sin(x)\cdot 0 - 1(\cos(x))}{\sin^2(x)} = \frac{-\cos(x)}{\sin^2(x)} = -\frac{\cos(x)}{\sin(x)}\cdot\frac{1}{\sin(x)} = -\cot(x)\csc(x)$$

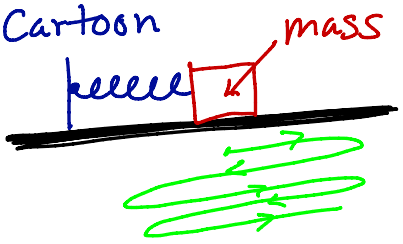
4. SUMMARY RULES:

Do you see the pattern?

$$\frac{d}{dx}[\tan(x)] = \sec^2(x) \qquad \frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$$

$$\frac{d}{dx}[\cot(x)] = -\csc^2(x) \qquad \frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$$

5. A mass on a spring vibrates horizontally on a smooth level surface. Its equation of motion is $x(t) = 8\sin(t)$, where t is in seconds and x is in centimeters.

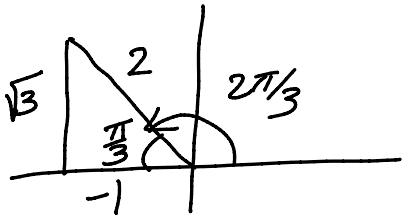


(a) Find the velocity and acceleration at time t.

$v(t) = x'(t) = 8\cos(t)$ ← cm/sec

$a(t) = x''(t) = -8\sin(t)$ ← cm/sec^2

Use this →



(b) Find the position, velocity, and acceleration of the mass at time $t = 2\pi/3$. In what direction is it moving at this time? Is it speeding up or slowing down?

$x(2\pi/3) = 8\sin(2\pi/3) = 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$ cm

$v(t) = 8\cos(2\pi/3) = 8\left(\frac{-1}{2}\right) = -4$ cm/sec

$a(t) = -8\sin(2\pi/3) = -8\left(\frac{\sqrt{3}}{2}\right) = -4\sqrt{3}$ cm/sec^2

Direction?
- to the left b/c $v < 0$

Speeding up/slowing?
- speeding up b/c v and a are both negative.