

SECTION 3-6: THE CHAIN RULE

1. Recall Two Versions of the Chain Rule

[A] $\frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x)$

[B] $y = f(u)$
 $u = g(x)$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

2. Understanding what the "formulas" in the book are trying to communicate:

Example:

From § 3.5,

$\frac{d}{dx} [\sec(x)] = \sec(x)\tan(x)$

In § 3.6 we see

$\frac{d}{dx} [\sec(g(x))] = \sec(g(x)) \cdot \tan(g(x)) \cdot g'(x)$

$\frac{d}{dx} [\sec(u)] = \sec(u)\tan(u) \cdot \frac{du}{dx}$

Just writing out the Chain Rule in a very specific case

$y = \sec(3x); y' = \sec(3x)\tan(3x) \cdot 3$

3. (Some additional independent practice) Find the derivatives.

(a) $f(x) = (\sec(3x) + \csc(2x))^5$

$f'(x) = 5(\sec(3x) + \csc(2x))^4 \cdot (3\sec(3x)\tan(3x) - 2\csc(2x)\cot(2x))$

(b) $g(x) = \frac{\cot(x^2+1)}{x^3+1}$

$g'(x) = \frac{(x^3+1)(-\csc^2(x^2+1)(2x)) - \cot(x^2+1)(3x^2)}{(x^3+1)^2}$

(c) $h(x) = (2x-1)^3(2x+1)^5$

$h'(x) = \underbrace{3(2x-1)^2(2)}_{f'} \underbrace{(2x+1)^5}_g + \underbrace{(2x-1)^3}_f \underbrace{5(2x+1)^4(2)}_{g'}$

If time permits, point out what would happen if the "3" was a "-3"

4. Find all x -values where the tangent to $f(x) = (x^2 - 4)^3$ is horizontal.

$f'(x) = 0$

$f'(x) = 3(x^2-4)^2(2x) = 6x(x^2-4)^2 = 0$

So $x=0$ or $x^2-4=0$.

$x^2-4=0$ when $x^2=4$ or $x=\pm 2$

Answer: $f(x)$ has a horizontal tangent when

$x = -2, 0, 2.$