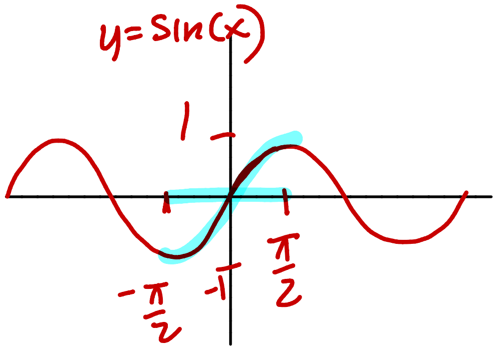
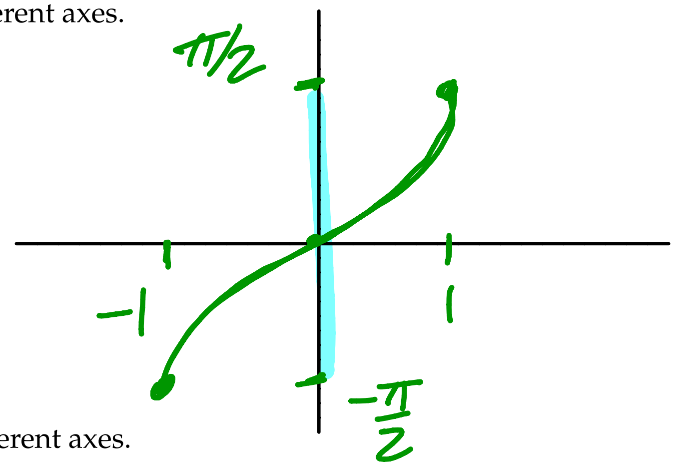
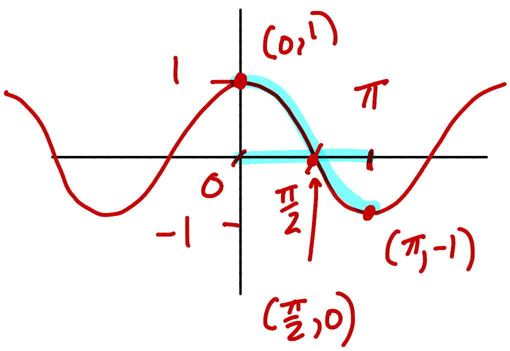
1. Motivating observation: Implicit differentiation can be used to find the derivatives of inverses.

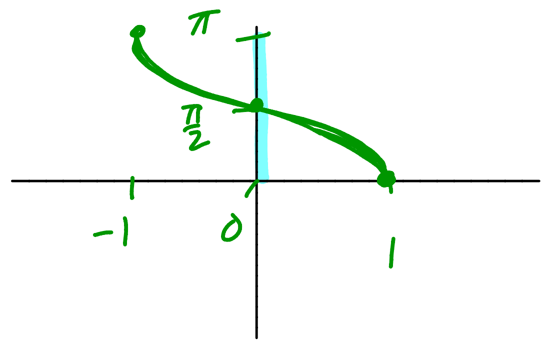
2. Graph $f(x) = \sin(x)$ and $f^{-1} = \sin^{-1}(x)$ on different axes.



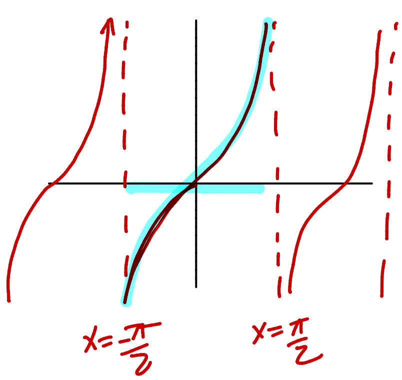


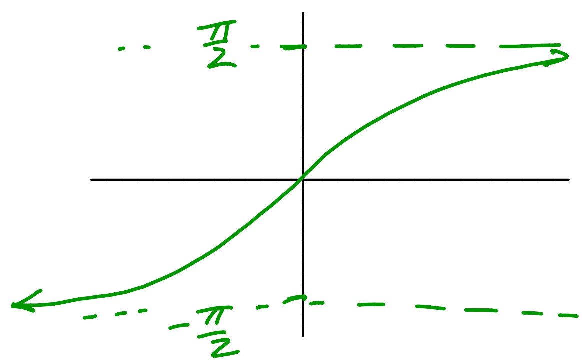
3. Graph $f(x) = \cos(x)$ and $f^{-1} = \cos^{-1}(x)$ on different axes.





4. Graph $f(x) = \tan(x)$ and $f^{-1} = \tan^{-1}(x)$ on different axes.





5. Formulas for the derivatives of inverse trigonometric functions.

$$\frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\arccos(x)\right] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\arctan(x)\right] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\mathrm{arcsec}(x)\right] = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left[\mathrm{arccot}(x)\right] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left[\mathrm{arccsc}(x)\right] = \frac{-1}{|x|\sqrt{x^2+1}}$$

Based on the graphs, are these derivative rules plausible?

6. Use the formulas on the previous page to find the derivatives of the functions below:

(a) $f(x) = \arcsin(2x)$

$$f'(x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot (2) = \frac{2}{\sqrt{1-4x^2}}$$

(b) $f(x) = 5x\arctan(\sqrt{x}) = 5x \arctan\left(x^{\frac{1}{2}}\right)$

$$f'(x) = 5 \cdot \arctan\left(x^{\frac{1}{2}}\right) + 5x\left(\frac{1}{1+(x^{\frac{1}{2}})^2}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) = 5\arctan(\sqrt{x}) + \frac{5x}{2\sqrt{x}(1+x)}$$

7. Use implicit differentiation to find the derivatives of the functions below.

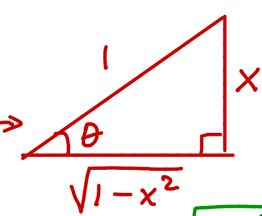
(a) $f(x) = \arcsin(x)$

$y = \arcsin(x)$ or $x = \sin(y)$

$$1 = \cos(y) \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\cos(\arcsin(x))} = \frac{1}{\sqrt{1-x^2}}$$

$\arcsin(x)$
$= \arcsin\left(\frac{x}{1}\right)$
$= \theta$



$$\cos(\arcsin(x)) = \cos(\theta) = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

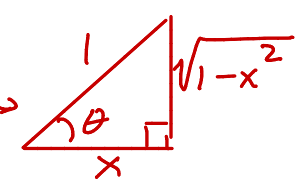
(b) $f(x) = \arccos(x)$

$y = \arccos(x)$ or $x = \cos y$

$$1 = -\sin(y) \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-1}{\sin(y)} = \frac{-1}{\sin(\arccos(x))} = \frac{-1}{\sqrt{1-x^2}}$$

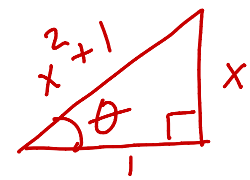
$\arccos(x)$
$= \arccos\left(\frac{x}{1}\right) = \theta$



$\sin(\arccos(x))$
$= \sin(\theta) = \sqrt{1-x^2}$

(c) $f(x) = \arctan(x)$

$\tan(\arctan(x))$
$= \tan(\theta) = x$



$y = \arctan(x)$ or $x = \tan(y)$

$$1 = \sec^2(y) \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2(y)} = \frac{1}{1+\tan^2(y)} = \frac{1}{1+[\tan(\arctan(x))]^2} = \frac{1}{1+x^2}$$