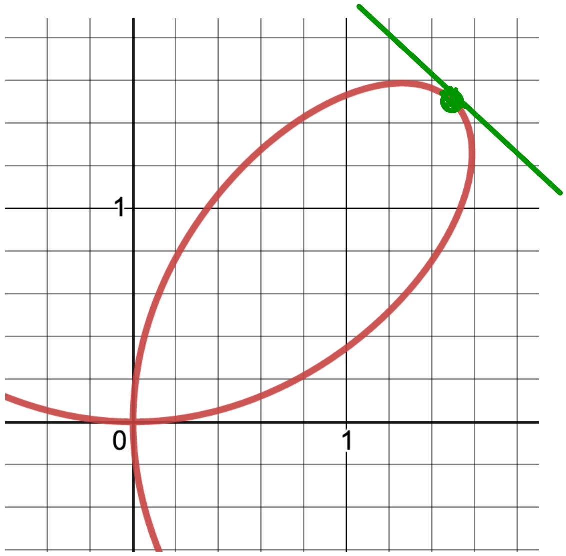
1. Motivating questions: How can we find slope of the tangent / velocity for a graph that looks like the one below?



Tangent line to $y^3 + x^3 = 3xy$ at $(3/2, 3/2)$?

$$3y^2\frac{dy}{dx} + 3x^2 = 3\cdot 1\cdot y + 3x\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y - 3x^2$$

$$\frac{dy}{dx}\left(3y^2 - 3x\right) = 3y - 3x^2$$

$$\frac{dy}{dx} = \frac{3y-3x^2}{3y^2-3x}\ ; \quad \left.\frac{dy}{dx}\right|_{(3/2,3/2)} = \frac{3\left(\frac{3}{2}\right) - 3\left(\frac{3}{2}\right)^2}{3\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right)} = -1 = m_{tan}\ ;$$

line: $y - \frac{3}{2} = -1\left(x - \frac{3}{2}\right)$ or

$y = 3 - x$

2. What is the derivative of: $(f(x))^3$?

$$3\left(f(x)\right)^2 \cdot f'(x)$$

3. Repeat question 2 above but with Leibniz notation. What is dy/dx for: $(y)^3$?

$$3y^2 \cdot \frac{dy}{dx}$$

That is, we are substituting:
$f(x) = y$
$f'(x) = \frac{dy}{dx}$

4. What is the derivative of $3xg(x)$?

$$3\cdot 1\cdot g(x) + 3x\cdot g'(x) = 3g(x) + 3xg'(x)$$

5. Repeat question 4 above but with Leibniz notation. What is dy/dx for: $3xy$?

$$3\cdot 1\cdot y + 3x\frac{dy}{dx} = 3y + 3x\frac{dy}{dx}$$

6. Find dy/dx for each expression below.

(a) $x^2 + y^3 = \cos(x) + \sin(y) + \pi/2$

$$2x + 3y^2 \frac{dy}{dx} = -\sin(x) + \cos(y) \cdot \frac{dy}{dx} + 0$$

$$3y^2 \frac{dy}{dx} - \cos(y)\frac{dy}{dx} = -2x - \sin(x)$$

$$\frac{dy}{dx}\left(3y^2 - \cos(y)\right) = -2x - \sin(x)$$

$$\frac{dy}{dx} = \frac{-2x - \sin(x)}{3y^2 - \cos(y)}$$

(b) $y\cos(x) + 2x = (y+1)^2$

$$\frac{dy}{dx} \cdot \cos(x) - y\sin(x) + 2 = 2(y+1) \cdot \frac{dy}{dx}$$

$$2 - y\sin(x) = 2(y+1)\frac{dy}{dx} - \cos(x)\frac{dy}{dx}$$

$$2 - y\sin(x) = \left(2(y+1) - \cos(x)\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2 - y\sin(x)}{2(y+1) - \cos(x)}$$

(c) $x + \tan(xy) = 5$

$$1 + \sec^2(xy)\left[1 \cdot y + x\frac{dy}{dx}\right] = 0$$

$$1 + y\sec^2(xy) + x\sec^2(xy)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-1 - y\sec^2(xy)}{x\sec^2(xy)}$$

7. For the equation $x^2 + xy + y^2 = 9$,

(a) Find the x intercept(s).

when $y = 0$. So $x^2 = 9$ or $x = \pm 3$

(b) Find the slope of the tangent lines at the x-intercepts.

Find dy/dx.

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$(x+2y)\frac{dy}{dx} = -2x - y \text{ ; So } \frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

at $(\pm 3, 0)$ $\frac{dy}{dx} = -2$

(c) Write the equations of the tangent lines at the x-intercepts.

$y = -2(x+3)$, $y = -2(x-3)$