Section 4.10 Antiderivatives

- (families of) antiderivatives
- indefinite integrals
- initial value problems

1. Find the (family of) antiderivatives for the following.

(a) $f(x) = 4x^3$ (b) $f(x) = 5\sin(x)$ (c) $f(x) = \frac{e^x}{4}$

$F(x) = x^4 + C$

$F(x) = -5\cos(x) + C$

$F(x) = \frac{1}{4}e^x + C$

(d) $f(x) = \sqrt{2}$ (e) $f(x) = \frac{1}{x}$ (f) $f(x) = 1 - x + e^x$

$F(x) = \sqrt{2}x + C$

$F(x) = \ln|x| + C$

$F(x) = x - \frac{1}{2}x + e^x + C$

2. Is $F(x) = x + xe^x$ is an antiderivative of $f(x) = (x+1)e^x + 1$? Show your answer is correct.

$$F'(x) = 1 + 1 \cdot e^x + xe^x$$
$$= 1 + (1+x)e^x$$
$$= (x+1)e^x + 1$$

They are the same so F is an antiderivative of f.

Function	Antiderivative
x^k $(k \neq -1)$	$x^{k+1}/k+1$
x^{-1} for all x	$\ln\|x\|$
1	x
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$

Function	Antiderivative
e^x	e^x
$1/(1+x^2)$	$\arctan(x)$
$\sec^2(x)$	$\tan(x)$
$\sec(x)\tan(x)$	$\sec(x)$
$1/\sqrt{1-x^2}$	$\arcsin(x)$

$-\frac{7}{4}+1=-\frac{7}{4}+\frac{4}{4}=\frac{-3}{4}$

3. Evaluate the integrals.

(a) $\int (x^{1/2}+x^{-7/4})dx = \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{-3/4}}{\frac{-3}{4}} + C = \frac{2}{3}x^{\frac{3}{2}} - \frac{4}{3}x^{-3/4} + C$

(b) $\int (8e^x + \sec^2(x))\,dx = 8e^x + \tan(x) + C$

(c) $\int \frac{x^2+x^{1/2}+1}{x^{1/2}}\,dx = \int \left(x^{3/2} + 1 + x^{-1/2}\right)dx = \frac{2}{5}x^{\frac{5}{2}} + x + 2x^{\frac{1}{2}} + C$

4. Is the equality in the box true or false? Explain. $\boxed{\int x\sec^2(x^2+1)\,dx = \tan(x^2+1)+C}$ False!

Check

$\frac{d}{dx}\left[\tan(x^2+1)\right] = \left(\sec^2(x^2+1)\right)(2x) = 2x\sec^2(x^2+1)$ Not the same

5. Solve the initial value problem if $f'(x) = x + e^x$ and $f(0) = 4$.

$f(x) = \frac{1}{2}x^2 + e^x + C$

$f(0) = \frac{1}{2}0^2 + e^0 + C = 4$

So $1 + C = 4$. So $C = 3$.

Answer:

$f(x) = \frac{1}{2}x^2 + e^x + 3$

6. A particle moving along the x-axis has acceleration $a(t) = 10\sin(t)$ measured in cm/s^2. Assume the particle as initial velocity $v(0) = 0$ and initial position $s(0) = 0$, find a function that models its velocity, $v(t)$, and its position $s(t)$.

$a(t) = 10\sin(t)$

$v(t) = \int a(t)\,dt = \int 10\sin(t)\,dt = -10\cos(t) + C$

$v(0) = -10\cos(0) + C = 0$. So $-10 + C = 0$. So $C = 10$.

So $v(t) = -10\cos(t) + 10$.

$s(t) = \int v(t)\,dt = \int(-10\cos(t) + 10)\,dt = -10\sin(t) + 10t + C$

$s(0) = -10\sin(0) + 10(0) + C = 0$. So $C = 0$.

$s(t) = -10\sin(t) + 10t$