1. The Second Derivative Test → Suppose $f'(a)=0$, f'' is continuous in interval containing a then:
   (i) $f(a)$ is a local min if $f''(a)>0$
   (ii) $f(a)$ is a local max if $f''(a)<0$
   (iii) If $f''(a)=0$, the test is inconclusive; we don't know if $f(a)$ is a local max, local min, or neither.

motivated by picture:

local min at $x=a$
concave up
$f''>0$

local max at $x=a$
concave down
$f''<0$

2. Use the Second Derivative Test to find the local extrema for $f(x)=-3x^5+5x^3$.

$f'(x)=-15x^4+15x^2=-15x^2(x^2-1)=-15x^2(x+1)(x-1)$

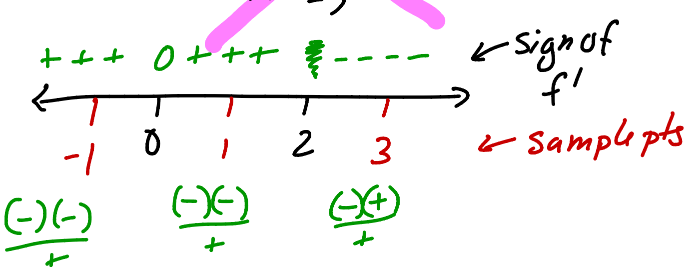
$f''(x)=-60x^3+30x=-30x(2x^2-1)$

C.pts: $x=0,+1,-1$; $f''(0)=0$, $f''(1)<0$, $f''(-1)>0$

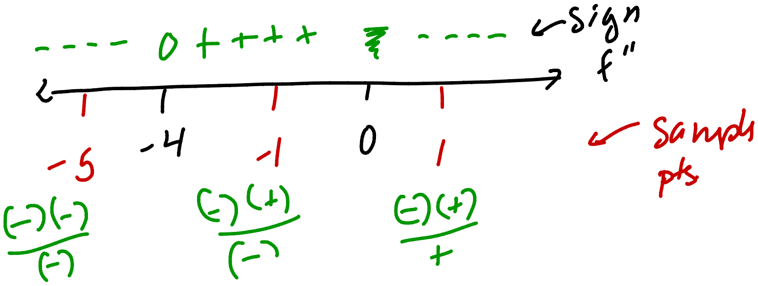
Conclusion of 2nd Der Test
$f(1)$ is a local max
$f(-1)$ is a local min
at $x=0$, the test tells us nothing. (In fact, $f(0)$ is neither max nor min.)

3. For the function $f(x)=\sqrt[3]{x}(8-x)$, determine (a) intervals where f is increasing/decreasing, (b) the locations of any local extrema (c) intervals where f is concave up / concave down (d) inflection points. Then use technology to confirm your answers.
   NOTE: $f'(x)=\dfrac{-4(x-2)}{3x^{2/3}}$ and $f''(x)=\dfrac{-4(x+4)}{9x^{5/3}}$

crit #'s: $x=2$, $x=0$



$f''=0$ when $x=-4$
f'' undefined when $x=0$



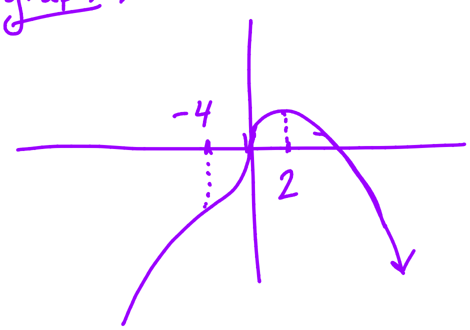
(a) $f(x)\uparrow$ on $(-\infty,2)$; $\downarrow$ on $(2,\infty)$

(b) $f(x)$ has a local max at $x=2$.
max value $f(2)=6(2)^{1/3}$.

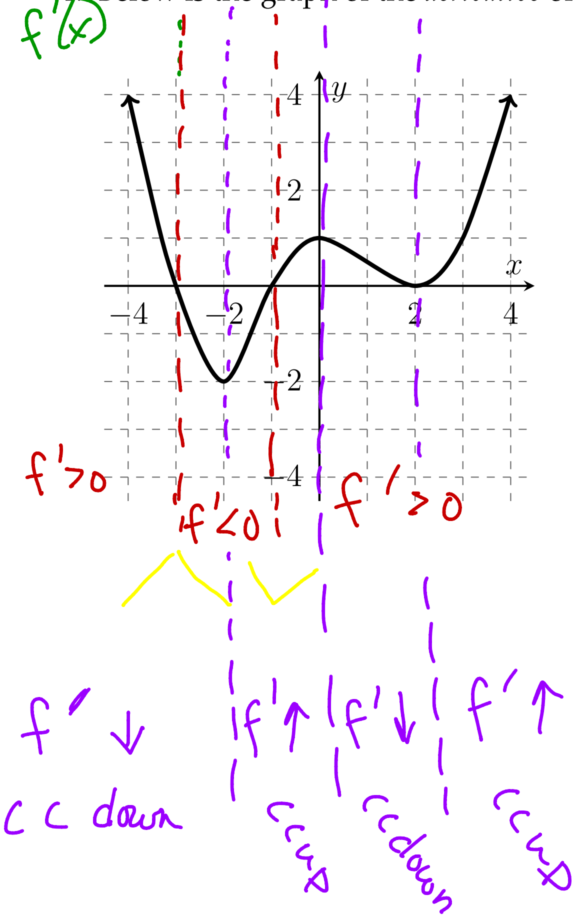
(c) f ccup on $(-4,0)$ and f ccdown on $(-\infty,-4)\cup(0,\infty)$.

(d) f has inflection points at
$(-4, f(-4))=(-4,-\sqrt[3]{4}(12))$
$(0,0)$

graph:



4. Below is the graph of the *derivative* of f, $f'(x)$. Use this graph to answer the questions.



(a) On what intervals is $f(x)$ increasing? decreasing?

f ↑ on $(-\infty,-3)\cup(-1,\infty)$, ↓ on $(-3,-1)$

(b) Determine the location of local extrema of f.

f has local min at $x=-1$, local max at $x=-3$

(c) On what intervals is $f(x)$ concave up? concave down?

f ccup on $(-2,0)\cup(2,\infty)$, cc down $(-\infty,-2)\cup(0,2)$

(d) Determine the location of any inflection points of f.

f has inflection pts at

$x=-2,0,2$

5. Sketch a graph that satisfies *all* of the properties below.

(a) $f(2)=f(4)=0$
(b) $f'(x)>0$ if $x<3$
(c) $f'(3)$ does not exist
(d) $f'(x)<0$ if $x>3$
(e) $f''(x)>0$ for $x\neq 3$.