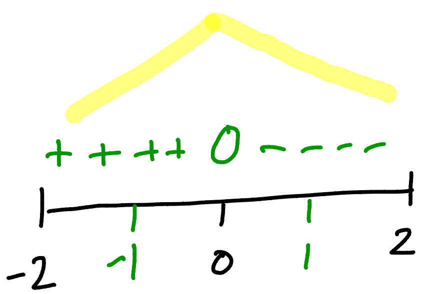
1. Given $g(x) = \sqrt{4-x^2}$, $f'(x) = \frac{-x}{\sqrt{4-x^2}}$, $f''(x) = \frac{-4}{(4-x^2)^{3/2}}$. Identify important features of $f(x)$ like: domain, asymptotes, local extrema, inflection points, and make a rough sketch.

domain: $4 - x^2 \geq 0$ or $4 \geq x^2$ or $-2 \leq x \leq 2$

So $\boxed{[-2,2]}$

asymptotes: none. (Note $\lim_{x\to\pm\infty} f(x)$ doesn't even make sense...)

$\uparrow, \downarrow$, and extrema: $f'(x) = 0$ when $x=0$. $f'(x)$ is undefined at endpoints $x = \pm 2$.



← sign of f'

$f'(-1) = \frac{+}{+} > 0$, $f'(1) = \frac{-}{+} < 0$

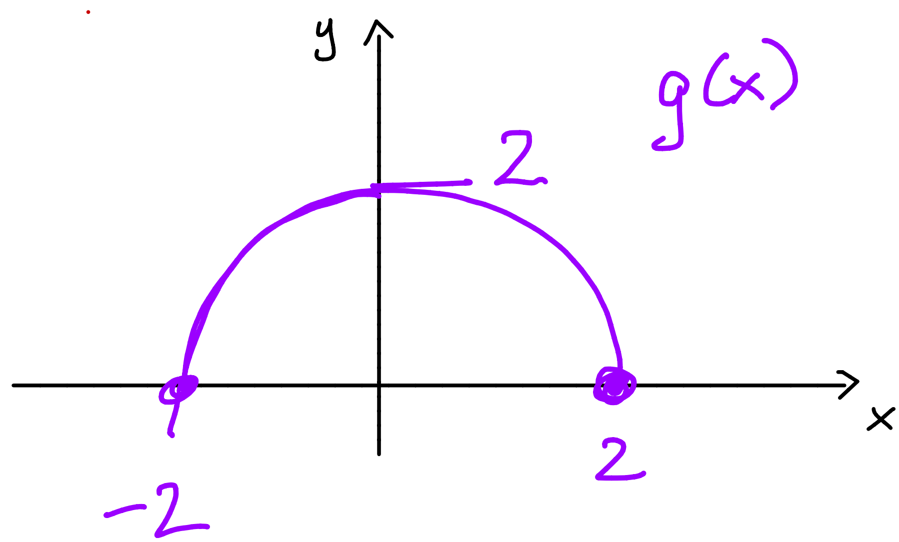
$f(x)$ is $\uparrow$ on $[-2,0)$, $\downarrow$ on $(0,2]$ and has an absolute maximum at $x=0$. The maximum value is $f(0) = \sqrt{4} = 2$.

concavity + i.pts: $f''(x) \leq 0$ for all x.

So $f(x)$ is concave down on $[-2,2]$ and has no inflection points.

plot some points for our graph:

x	f(x)
-2	0
0	2
2	0



Yes, $g(x)$ is the top-half of a circle. Do you see why??

2. Given $f(x) = x^2 + \frac{4}{x^2}$, $f'(x) = 2x - \frac{8}{x^3}$, $f''(x) = 2 + \frac{24}{x^2}$. Identify important features of $f(x)$ like: domain, asymptotes, local extrema, inflection points, and make a rough sketch.

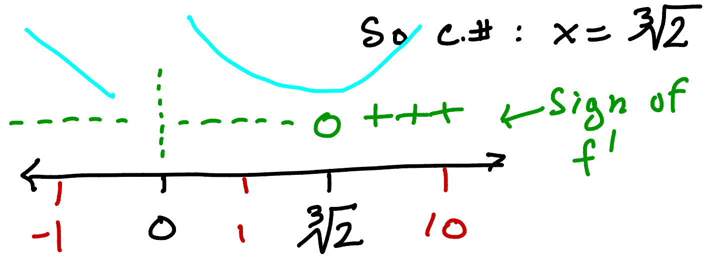
domain : $(-\infty, 0) \cup (0, \infty)$

asymptotes : h.a.: none (b/c $\lim_{x \to \pm\infty} f(x) = \pm\infty$)

v.a.: $x=0$. $\lim_{x \to 0^+} x^2 + \frac{4}{x} = +\infty$, $\lim_{x \to 0^-} x^2 + \frac{4}{x} = -\infty$

↑, ↓, extrema : $f'(x) = 0$ when $2x - \frac{4}{x^2} = 0$ or $2x = \frac{4}{x^2}$ or $x^3 = 2$

So c.#: $x = \sqrt[3]{2}$



$f'(-1) = -2 - 4 < 0$; $f'(1) = 2 - 4 < 0$

$f'(10) = 20 -$ tiny# > 0.

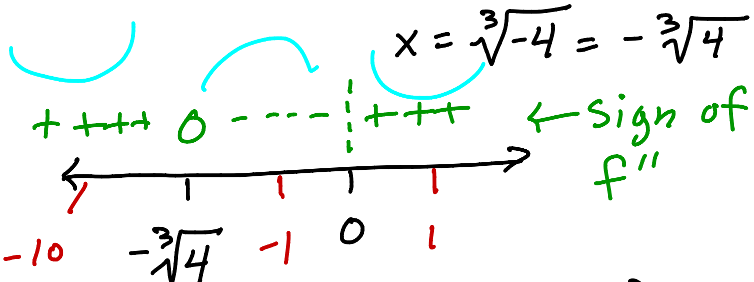
$f(x)$ is ↑ on $(\sqrt[3]{2}, \infty)$ and ↓ on $(-\infty, 0) \cup (0, \sqrt[3]{2})$.

$f(x)$ has a local min at $x = \sqrt[3]{2}$ and no local max.

No absolute extrema at all (b/c limits at $\pm\infty$).

concavity : $f''(x) = 0$ when $2 + \frac{8}{x^3} = 0$ or $2x^3 = -8$ or

$x = \sqrt[3]{-4} = -\sqrt[3]{4}$.



$f''(-10) = 2 -$ tiny# > 0

$f''(-1) = 2 - 8 < 0$,

$f''(1) = 2 + 8 > 0$.

f is ccup on $(-\infty, -\sqrt[3]{4}) \cup (0, \infty)$ and ccdown $(-\sqrt[3]{4}, 0)$.

f has an inflection point at $x = -\sqrt[3]{4}$. (Not at $x=0$ since $x=0$ is not in the domain)

Put it all together to sketch the graph