1. Limits at Infinity: In plain English, what should the symbols below mean?

$\lim_{x\to\infty} f(x) = L$ As x gets larger & larger, f(x) gets closer & closer to the y-value L.

$\lim_{x\to-\infty} f(x) = L$ As x gets smaller & smaller, f(x) gets closer & closer to the y-value L.

2. Three Principles (a is a constant) and a Strategy

- If a is a constant, then $\lim_{x\to\pm\infty} ax = \pm\infty$ (just have to check the sign)
- $\lim_{x\to\pm\infty} \frac{1}{x} = 0$
- If $\lim_{x\to\pm\infty} f(x) = a$ and $\lim_{x\to\pm\infty} g(x) = \pm\infty$, then $\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} = 0$
- Strategy: Divide numerator and denominator by the highest power of x in the denominator.

3. Use the Principles to evaluate the limits below. Then, use your calculator to confirm your answer is correct.

(a) $\lim_{x\to\infty} \frac{(2x^2 - x)\cdot\frac{1}{x^2}}{(3x - 5x^2)\cdot\frac{1}{x^2}} = \lim_{x\to\infty} \frac{2 - \frac{1}{x}}{\frac{3}{x} - 5} = \frac{2}{-5} = -\frac{2}{5}$ ($\frac{1}{x}\to 0$, $\frac{3}{x}\to 0$)

Check: $\frac{2(1000)^2 - (1000)}{3(1000) - 5(1000)^2} = -0.4000400\ldots$ ✓

(b) $\lim_{x\to\infty} \frac{(2x^3 - x)\cdot\frac{1}{x^2}}{(3x - 5x^2)\cdot\frac{1}{x^2}} = \lim_{x\to\infty} \frac{2x - \frac{1}{x}}{\frac{3}{x} - 5} = \infty$ ($2x\to\infty$, $\frac{1}{x}\to 0$, $\frac{3}{x}\to 0$, $-5\to -5$)

(c) $\lim_{x\to\infty} \frac{(3x + \sin(x))\left(\frac{1}{x}\right)}{(x)\left(\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{3 + \frac{\sin(x)}{x}}{1} = 3$

$\frac{\sin(x)}{x} \to 0$ b/c $-1 \le \sin(x) \le 1$

(d) $\lim_{x\to-\infty} \frac{2x+1}{\sqrt{x^2+1}}$ (Pay attention to the sign here!)

my answer

$\lim_{x\to-\infty} \frac{(2x+1)\cdot\frac{1}{|x|}}{\left(\sqrt{x^2+1}\right)\cdot\frac{1}{|x|}} = *$

crucial facts:

- $\sqrt{x^2} = |x|$
- $\lim_{x\to-\infty} \frac{x}{|x|} = -1$

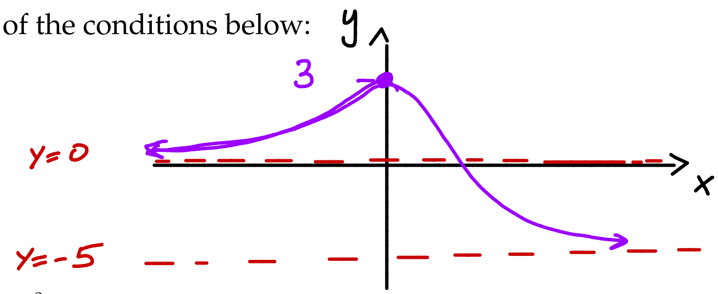
$* = \lim_{x\to-\infty} \frac{\frac{2x}{|x|} + \frac{1}{|x|}}{\sqrt{\frac{x^2+1}{x^2}}} = \lim_{x\to-\infty} \frac{\frac{2x}{|x|} + \frac{1}{|x|}}{\sqrt{1 + \frac{1}{x^2}}} = \frac{-2+0}{\sqrt{1+0}} = -2$

4. Fill in the blanks.

- If $\lim_{x\to\infty} f(x) = L$, then $\underline{\quad y = L \quad}$ is an asymptote of the graph of $f(x)$.
- If $\lim_{x\to-\infty} f(x) = L$, then $\underline{\quad y = L \quad}$ is an asymptote of the graph of $f(x)$.

5. Sketch a graph of a function $g(x)$ that satisfies all of the conditions below:

- it's continuous on $(-\infty, \infty)$
- it has an absolute maximum of 3 at $x = 0$
- $\lim_{x\to\infty} g(x) = -5$
- $\lim_{x\to-\infty} g(x) = 0$.



6. Given $f(x) = \frac{x^2}{x^2+1}$, $f'(x) = \frac{2x}{(x^2+1)^2}$, $f''(x) = \frac{-2(3x^2-1)}{(x^2+1)^3}$. Identify important features of $f(x)$ like: domain, asymptotes, local extrema, inflection points, and make a rough sketch.

domain: $(-\infty, \infty)$ asymptotes: v.a. none; h.a at $y=1$ $\left(\lim_{x\to\pm\infty} \frac{x^2}{x^2+1} = 1\right)$

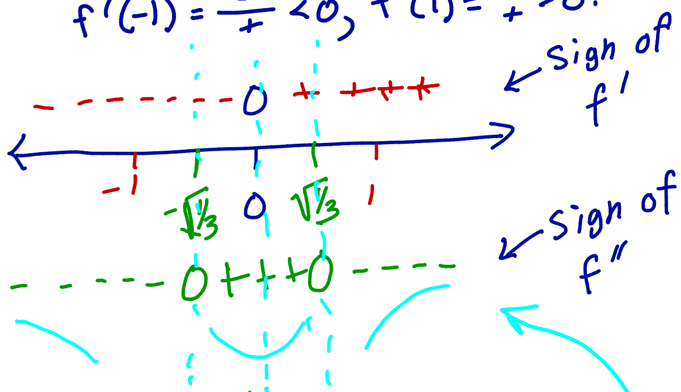
$\uparrow, \downarrow$, extrema: $f'(x) = 0$ when $x = 0$.

$f'(-1) = \frac{-}{+} < 0$, $f'(1) = \frac{+}{+} > 0$.

$f(x)$ is $\downarrow$ on $(-\infty, 0)$ and $\uparrow$ on $(0, \infty)$.

$f(x)$ has an absolute min at $x=0$.

and no local or abs max.



concavity $f''(x) = 0$ when

$3x^2 - 1 = 0$

$x^2 = \frac{1}{3}$

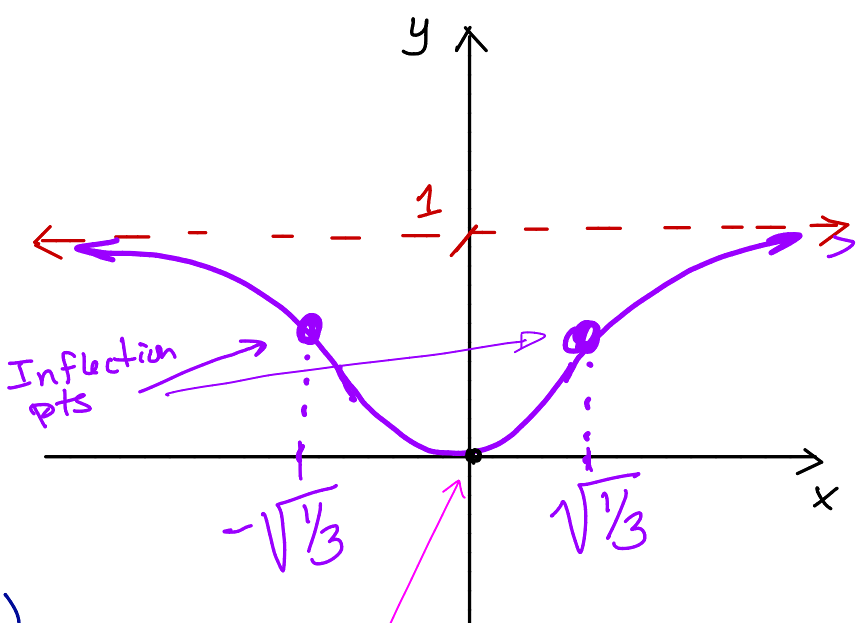
$x = \pm\sqrt{1/3}$

$f''(-1) = \frac{- \cdot +}{+} < 0$, $f''(0) = \frac{- \cdot -}{+} > 0$

$f''(1) = \frac{- \cdot +}{+} < 0$

$f(x)$ is cc up on $(-\sqrt{1/3}, \sqrt{1/3})$

and cc down on $(-\infty, -\sqrt{1/3}) \cup (\sqrt{1/3}, \infty)$



absolute minimum of 0 at $x=0$