

REVIEW SECTION 4.8 L'HÔPITAL'S RULE

1. When can you use L'Hôpital's Rule *directly*?

When the limit has the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

2. Evaluate the following limits.

$$(a) \lim_{x \rightarrow 0} \frac{\cos(\pi x) - 1}{x^2} \stackrel{\textcircled{H}}{=} \lim_{x \rightarrow 0} \frac{-\pi \sin(\pi x)}{2x} \stackrel{\textcircled{H}}{=} \lim_{x \rightarrow 0} \frac{-\pi^2 \cos(\pi x)}{2} = -\frac{\pi^2}{2}$$

form $\frac{0}{0}$ form $\frac{0}{0}$

$$(b) \lim_{x \rightarrow -\infty} \frac{e^x}{x} = 0$$

As $x \rightarrow -\infty$, $e^x \rightarrow 0$ and $x \rightarrow -\infty$.

So $\frac{e^x}{x} \rightarrow 0$.

$$(c) \lim_{x \rightarrow \infty} \left(1 - \frac{4}{x}\right)^x = \boxed{e^{-4}}$$

change problem

$$\lim_{x \rightarrow \infty} x \ln(1 - 4x^{-1}) = \lim_{x \rightarrow \infty} \frac{\ln(1 - 4x^{-1})}{x^{-1}} \stackrel{\textcircled{H}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{1-4x^{-1}} \cdot 4x^{-2}}{-x^{-2}}$$

$$= \lim_{x \rightarrow \infty} \frac{-4}{1-4x^{-1}} = -4$$