1. L'Hôpital's Rule says:

If $\lim_{x\to a} \frac{f(x)}{g(x)} = \begin{cases} 0/0 \\ \infty/\infty \end{cases}$, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$ provided the second limit exists or is $\pm\infty$.

2. Example $0/0$.

(a) $\lim_{x\to 2} \frac{x^2-4}{x^2-2x} \overset{H}{=} \lim_{x\to 2} \frac{2x}{2x-2} = \frac{2\cdot 2}{2\cdot 2-2} = \frac{4}{2} = 2$

plug in $= \frac{2^2-4}{2^2-4} = \frac{0}{0}$ form $\frac{0}{0}$

* Could have worked this by factoring! $\lim_{x\to 2} \frac{(x-2)(x+2)}{x(x-2)} = \lim_{x\to 2} \frac{x+2}{x} = \frac{2+2}{2} = 2$

You. $\boxed{\text{(b)}}$ $\lim_{x\to 0} \frac{\sin(4x)}{3e^{3x}-3} \overset{H}{=} \lim_{x\to 0} \frac{4\cos(4x)}{9e^{3x}} = \frac{4\cos(0)}{9e^0} = \frac{4\cdot 1}{9\cdot 1} = \frac{4}{9}$

form $\frac{0}{0}$

3. Example ∞/∞.

(a) $\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}} \overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{2\sqrt{x}}{x} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0$

form $\frac{\infty}{\infty}$

(b) $\lim_{x\to\infty} \frac{2e^x+1}{1-3e^x} \overset{H}{=} \lim_{x\to\infty} \frac{2e^x}{-3e^x} = \lim_{x\to\infty} \frac{-2}{3} = \frac{-2}{3}$

form $\frac{\infty}{-\infty}$

4. Example $0\cdot\infty$.

(a) $\lim_{x\to\infty} x\sin(\frac{\pi}{x})$ $\overset{\text{rewrite as } \frac{0}{0}}{=}$ $\lim_{x\to\infty} \frac{\sin(\pi x^{-1})}{x^{-1}} \overset{H}{=} \lim_{x\to\infty} \frac{\cos(\pi x^{-1})(-\pi x^{-2})}{-x^{-2}} = \lim_{x\to\infty} \pi\cos(\frac{\pi}{x})$

$\frac{\pi}{x}\to 0$, $\sin(\frac{\pi}{x})\to 0$. $x\to\infty$.

form $\infty\cdot 0$

form $\frac{0}{0}$

cancel

$= \pi\cos(0) = \pi$.

$*$ use $x = \frac{1}{\frac{1}{x}}$

or $x = x^1 = x^{(-1)(-1)} = (x^{-1})^{-1}$

(b) $\lim_{x\to 0+} x\ln(x)$ $= \lim_{x\to 0^+} \frac{\ln(x)}{x^{-1}} \overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-x^{-2}} = \lim_{x\to 0^+} \frac{-x^2}{x}$

form $0\cdot(-\infty)$ rewrite

form $\frac{-\infty}{\infty}$

$= \lim_{x\to 0^+} -x = -0$

$= 0$.

leave space for answer.

5. Example 1^∞ or 0^0 or ∞^0

(a) $\lim_{x\to 0+} (1+x)^{1/x}$ $= \boxed{e^1 = e}$

1^∞

change problem:

$\lim_{x\to 0^+} \frac{1}{x}\ln(1+x) = \lim_{x\to 0^+} \frac{\ln(1+x)}{x} \overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{1+x}}{1} = \frac{\frac{1}{1+0}}{1} = 1$

form $\frac{0}{0}$

(b) $\lim_{x\to 0+} x^{\sin(x)}$ $= \boxed{e^0 = 1}$

0^0

change problem

$\lim_{x\to 0^+} \sin(x)\cdot\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{(\sin(x))^{-1}} \overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-(\sin(x))^{-2}(\cos(x))} = \lim_{x\to 0^+} \frac{-\sin^2 x}{x\cos(x)}$

form $\frac{0}{0}$

form $0\cdot(-\infty)$

form $\frac{-\infty}{\infty}$

$\overset{H}{=} \lim_{x\to 0^+} \frac{-2\sin(x)\cos(x)}{x(-\sin(x))+1\cdot\cos(x)} = \frac{-2\cdot 0\cdot 1}{0(0)+1\cdot 1}$

$= \frac{0}{1} = 0$