SECTION 4.9 NEWTON'S METHOS

1. Why would you want to solve $f(x) = 0$?

$f=0$ indicates where f may change signs
If $F' = f = 0$, then solutions may indicate locations of maxs/mins
If $F'' = f = 0$, then solutions may indicate locations where concavity changes

2. You are going to produce the *iterative* formula that is Newton's Method.

   (a) Find the equation of the line tangent to $f(x)$ at $x = x_1$. (Assume $f'(x_1) \neq 0$.)

   point: $(x_1, f(x_1))$
   slope: $m = f'(x_1)$
   line: $\boxed{y - f(x_1) = f'(x_1)(x - x_1)}$

   (b) Determine the x-value where the tangent line from part (a) intersects the x-axis. Call this x-value x_2.

   Intersecting x-axis $\equiv$ $y=0$.
   Set $y=0$.
   Solve for $x = x_2$.

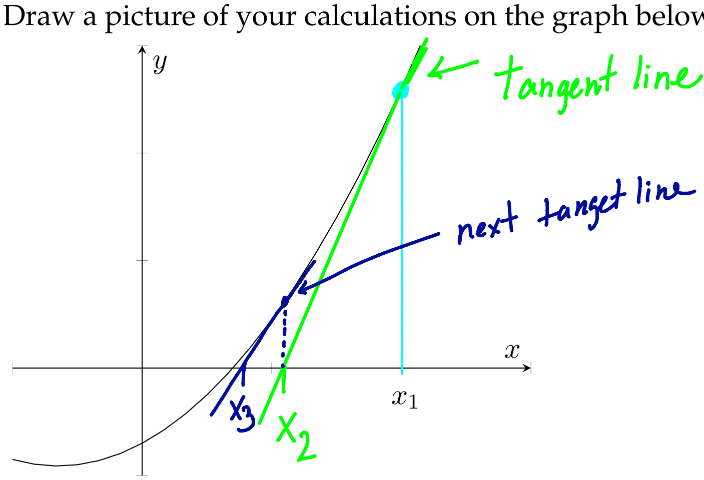
   $$0 - f(x_1) = f'(x_1)(x_2 - x_1)$$

   Solve for x_2

   $$\frac{-f(x_1)}{f'(x_1)} = x_2 - x_1$$

   $$x_1 - \frac{f(x_1)}{f'(x_1)} = x_2$$

   (c) Draw a picture of your calculations on the graph below.



   (d) Given a guess x_n, write the formula for how to get a better guess, x_{n+1}.

   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

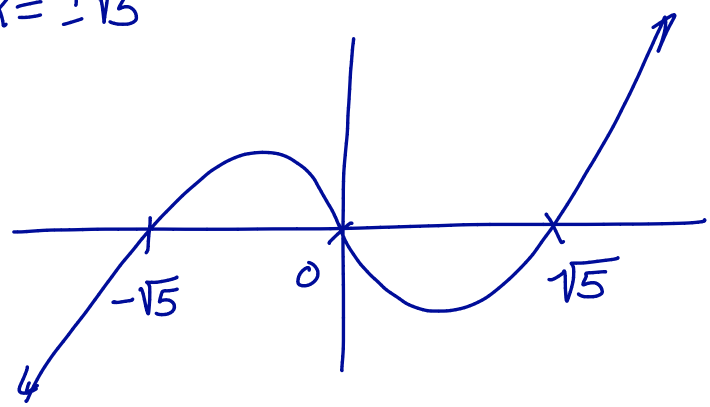
3. $\boxed{\text{MODEL PROBLEM:}}$ Let $f(x) = x^3 - 5x$. $\quad f'(x) = 3x^2 - 5$

(a) Factor $f(x)$, find its roots algebraically, and sketch its graph.

$f(x) = x(x^2-5) = x(x-\sqrt{5})(x+\sqrt{5}) = 0$

roots $x = 0,\ x = \pm\sqrt{5}$

$\sqrt{5} = 2.2360679775$



(b) Assume you couldn't factor the function and you wanted to find its positive root. What would be a reasonable first guess and why?

$x_0 = 2$ or $x_0 = 2.5$

(c) Using a first guess of $x_1 = 3$, calculate 3 iterations of Newton's method $\quad f'(x) = 3x^2 - 5$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 3 - \dfrac{3^3 - 5\cdot 3}{3\cdot 3^2 - 5} = 2.4545454545 = x_2$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 2.262153775 = x_3$

$x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)} = 2.236512357 = x_4$

$2.2360679777\ldots \approx \sqrt{5}$

Calculations via Google Sheets

(d) How close is your estimate of the root, x_3, to the actual root?

It's correct to the $\frac{1}{1000}$ decimal position.

As an aside: $x_5 = 2.236068110$
$x_6 = 2.236067977$