SECTION 5.3: THE FUNDAMENTAL THEOREM OF CALCULUS (DAY 2)

1. The Fundamental Theorem of Calculus (part 1):

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

2. Find the derivative of each function below.

(a) $g(x) = \int_{-1}^{x} t^2 e^t \, dt$

$g'(x) = x^2 e^x$

(b) $h(x) = \int_0^{x^2+1} \sin(t) \, dt$

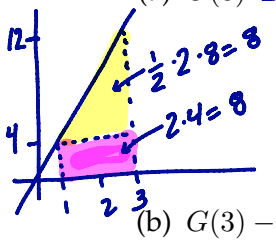
$\frac{dh}{dx} = \sin(x^2+1)(2x) = 2x\sin(x^2+1)$

3. Let $f(x) = 4x$. Find two different antiderivatives of $f(x)$. Call them $F_1(x)$ and $F_2(x)$.

$F_1(x) = 2x^2$, $F_2(x) = 2x^2+1$

4. Let $G(x) = \int_1^x 4t \, dt$. Evaluate the following:

(a) $G(3) = \int_1^3 4t\,dt = 8+8 = 16$



(b) $G(3) - G(1) = \int_1^3 4t\,dt - \int_1^1 4t\,dt = 16 - 0 = 16$

(c) $F_1(3) - F_1(0) = 2(3)^2 - 2(1)^2 = 18 - 2 = 16$

(d) $F_2(3) - F_2(0) = (2\cdot 3^2+1) - (2\cdot 1^2+1) = 2\cdot 3^2 - 2(1)^2 + 1 - 1$
$= 2\cdot 3^2 - 2(1)^2 = 16$

5. The Fundamental Theorem of Calculus (part 2):

$\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b)-F(a)$ where $F'(x)=f(x)$ provided $f(x)$ is continuous on $[a,b]$.

6. Evaluate the integrals.

(a) $\int_0^{\pi} \sin(x)\,dx$

$= -\cos(x)\Big]_0^{\pi}$

$= -\cos(\pi) - (-\cos(0)) = +1+1 = 2$

(b) $\int_{-1}^{3} x + e^x\,dx = \frac{1}{2}x^2 + e^x\Big]_{-1}^{3}$

$= \left(\frac{1}{2}\cdot 3^2 + e^3\right) - \left(\frac{1}{2}(-1)^2 + e^{-1}\right)$

$= \frac{9}{2} + e^3 - \frac{1}{2} - e^{-1} = 4 + e^3 + \frac{1}{e}$

7. Find the average value of $f(x) = x^2$ over the interval $[0,3]$.

$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$; $f_{ave} = \frac{1}{3}\int_0^3 x^2\,dx = \frac{1}{3}\left(\frac{1}{3}x^3\right]_0^3 = 3$

8. Assume the velocity of an object thrown directly up into the air is given by $v(t) = 20 - 9.8t$ where v is measured in meters per second and t is measured in seconds.

(a) Evaluate $\int_0^1 v(t)\,dt$

$\int_0^1 v(t)\,dt = \int_0^1 (20-9.8t)\,dt = 20t - 4.9t^2\Big]_0^1 = 20-4.9 = 15.1$ m

(b) Evaluate $\int v(t)\,dt$

$\int (v(t))\,dt = 20t - 4.9t^2 + C$

(c) Explain why you do not have enough information to find the height of the object exactly?

We don't know the height of the object when it was thrown.

(d) Explain, in the context of the problems what part (a) and part (b) represent.

(a) The change in height of the object in the first second. That is, the object is 15.1 m higher at 1 second that it was at 0 sec.

(b) An equation for position (or height) but w/o more information, we can't find it exactly.