SECTION 5.3: THE FUNDAMENTAL THEOREM OF CALCULUS (DAY 2)

1. The Fundamental Theorem of Calculus (part 1):

2. Find the derivative of each function below.

   (a) $g(x) = \int_{-1}^{x} t^2 e^t \, dt$

   (b) $h(x) = \int_{0}^{x^2+1} \sin(t) \, dt$

3. Let $f(x) = 4x$. Find two different antiderivatives of $f(x)$. Call them $F_1(x)$ and $F_2(x)$.

4. Let $G(x) = \int_1^x 4t \, dt$.

   (a) What do $G(x)$, $F_1(x)$ and $F_2(x)$ all have in common?

   (b) Find $G(1)$, $F_1(1)$ and $F_2(1)$. You will have to find $G(3)$ by geometry.

   (c) Find $G(3)$, $F_1(3)$ and $F_2(3)$. You will have to find $G(3)$ by geometry.

   (d) Using your answers above, find $G(3) - G(1)$ and explain what it means geometrically about the curve $y = 4x$.

   (e) Find $F_1(3) - F_1(1)$ and $F_2(3) - F_2(1)$.

   (f) What do parts (d) and (e) indicate about how you can calculate the (signed) area under a curve $f(x)$ on an interval $[a, b]$?

5. The Fundamental Theorem of Calculus (part 2):

6. Evaluate the integrals.

   (a) $\int_0^{\pi} \sin(x)\, dx$

   (b) $\int_{-1}^{3} x + e^x\, dx$

7. Find the average value of $f(x) = x^2$ over the interval $[0, 3]$.

8. Assume the velocity of an object thrown directly up into the air is given by $v(t) = 20 - 9.8t$ where v is measured in meters per second and t is measured in seconds.

   (a) Evaluate $\int_0^1 v(t)\, dt$

   (b) Evaluate $\int v(t)\, dt$

   (c) Explain why you do not have enough information to find the height of the object exactly?

   (d) Explain, in the context of the problems what part (a) and part (b) represent.