

Logistics**Topics****Section 2.1**

Secant lines and tangent lines. Average velocity and instantaneous velocity. Average rate of change and instantaneous rate of change.

Example: Sketch the graph $y = x^3 + 1$. Find the secant line between the points on the graph where $x = 1$ and $x = 3$. Sketch the secant line on the graph. Find an equation of the tangent line to the graph at $x = 1$ and sketch it on the graph. (NOTE: We get to answer the second part of this question using our knowledge of the derivative!)

Section 2.2

One-sided and two-sided limits from a graph. Vertical asymptotes and limits.

Example: Sketch a graph with *all* of the following properties:

- $f(x)$ is defined for all real numbers. (ie its domain is $(-\infty, \infty)$).
- $\lim_{x \rightarrow 1^-} f(x) = 0$, $\lim_{x \rightarrow 1^+} f(x) = 4$, $f(1) = 4$
- $\lim_{x \rightarrow 3} f(x) = 4$, $f(4) = -1$.
- $\lim_{x \rightarrow -1^-} f(x) = \infty$

Section 2.3

Evaluating limits algebraically.

Example: Evaluate $\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{9 - x}$ and $\lim_{x \rightarrow 1/2^+} \frac{4x^2 - 18x}{2x - 1}$

Section 2.4

Continuity. From a graph, determine where a graph is or is not continuous. From an algebraic description of a function, determine where a function is or is not continuous. Be able to explain why a function is not continuous at a point. The Intermediate Value Theorem.

Example: Look at your graph from Section 2.2. Where does it fail to be continuous and why? Where are the functions $f(x) = \frac{3 - \sqrt{x}}{9 - x}$ and $g(x) = \frac{4x^2 - 18x}{2x - 1}$ continuous?

Section 3.1

The relationship between secant lines and the derivative.

Example: Explain what the expression $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ means in terms of secant lines, tangent lines and the derivative. Draw a picture to illustrate your idea.

Section 3.2

The derivative as a function. The formal definition of the derivative. The relationship between the graph of $f(x)$ and the graph of $f'(x)$.

Example: Sketch the derivative of your graph from the Section 2.2 example. Use the definition of the derivative to find $f'(x)$ for $f(x) = 1/x^2$.

Section 3.3

Derivative rules: power, constant, sum/difference, product, quotient.

Example: Find the derivative of $y = 2x^{0.05} - \frac{x}{10}$ and $f(x) = \frac{x^3}{1-x}$

Section 3.4

The derivative as a rate of change. Interpretations of the derivative. Velocity and acceleration.

Example: Assume the distance traveled by a snow machine on a straight trail is given by $s(t)$ where t is in hours starting at 12 noon and s is in miles. Interpret $s'(4) = 10$. Interpret $s(4) - s(0)$. Interpret $(s(4) - s(1))/(4 - 1)$. Using the fact that that $s''(4) = -1.2$, estimate $s'(4.5)$.

Sections 3.5-3.9

Techniques and rule for taking derivative

Section 4.1

Related Rate Problems. All of these problems are word problems asking for a rate of change of some quantity with respect to time.

Example: An airplane is flying overhead at a constant elevation of 4000 feet as it passes directly over a man standing on the ground. If the plane is flying at a speed of 600 feet per second, how fast is the plane moving away from the man 5 seconds after it passes over his head? Assume the plane is flying in a straight line.

Section 4.2

Linear Approximations and Differentials. These problems ask you to find the linear approximation or differential of a function for particular values and then use these things (the linear approximation or differential) to estimate other things.

Example: Find the linear approximation of $f(x) = 5 \sin(x)$ when $a = 0$ and use it to estimate $5 \sin(-0.1)$

Example: Find the differential of $f(x) = 4\sqrt{x}$ when $x = 9$ and use it to estimate how much f will change if x changes from 9 to 9.01

Section 4.3

Maxima and minima. Absolute and local. Critical points. These problems are of two types: Finding ABSOLUTE extrema on closed-bounded intervals and finding local extrema in general.

Example: Find the absolute maximum and the absolute minimum of $f(x) = x^2 - 3x^{2/3}$ on $[0, 8]$.

Example: Identify any local extrema of $y = x^2 - \frac{1}{x^2}$.

Section 4.5

Derivatives and the Shape of a Graph. These problems ask you to use f' and f'' to determine when the original function, f , is increasing or decreasing, concave up or concave down, has extrema, has inflection points, and to draw sophisticated graphs.

Example: Draw some not-too-complicated graph. Now assume it is f' . What can you say about the graph of f ?

Example: If $f' > 0$ for $x > 0$, $f' < 0$ for $x < 0$, $f'' > 0$ for $-2 \leq x \leq 2$ and $f'' < 0$ for $x < -2$ and for $x > 2$, sketch f .

Section 4.6

Limits at Infinity and Asymptotes. The problems either ask you to evaluate a limit as $x \rightarrow \pm\infty$ or ask to find and justify the existence of a horizontal asymptote.

Example: Determine if the graph of $f(x) = \frac{3x^3 - e^x}{2x^3}$ has a horizontal asymptote. Justify your answer.

Section 4.7

Optimization Problems. These are word problems where you are asked to maximize or minimize some quantity. Crucial steps here include

- (a) identify the quantity to be maximized/minimized,
- (b) write the quantity from part (a) as a function of one variable,
- (c) identify the domain of the function from part (b),
- (d) take derivative and find critical points for function from part (b),
- (e) check/justify that your cp actually corresponds to a max/min,
- (f) answer the question.

Example: Go work problems from old midterms.

Section 4.8

L'Hopital's Rule. Be able to use L'Hopital's Rule to evaluate limits of a variety of indeterminate forms.

Example: Evaluate $\lim_{x \rightarrow 0^+} x \ln(x^4)$

Section 4.10

Antiderivatives and Initial Value Problems

Example: Evaluate $\int (\frac{3}{\sqrt{x}} - \csc^2(x)) dx$

Example: If an object has acceleration $a(t) = x + \sin(x)$, find its velocity equation assuming $v(0) = 10$.

Section 5.1

Approximating areas. Use rectangles with left- or right-hand endpoints to estimate the area under a curve.

Example: Use L_8 (ie eight rectangles with left-hand endpoints) to estimate the area under $y = \sqrt{x}$ on the interval $[0, 4]$. No need to get a decimal approximation. (!!)

Section 5.2

The Definite Integral as Signed Area under a Curve.

Example: Sketch the graph of $y = 10 - 5x$. Use this graph to evaluate $\int_1^6 (10 - 5x) dx$.

Section 5.3

The Fundamental Theorem of Calculus, parts I and II.

Example: Find the derivative of the function $F(x) = \int_1^{\cos(x)} (1 - t^2) dt$

Example: Evaluate $\int_1^5 \frac{x}{1+x^4} dx$

Section 5.4

The Net Change Theorem

Example: If $v(t)$ is the velocity of a car along a straight road in miles per hour, interpret the meaning of $\int_1^5 v(t) dt = -20$. Assume 1 and 5 are measured in hours.

Section 5.5

The method of substitution. (See the second example from Section 5.3 above.)

Sections 5.6-5.7

More integration formulas including those of exponential functions, logarithms, and inverse trigonometric functions.