REVIEW FOR FINAL EXAM

Topics from Chapter 5:

- §5.1 & 5.2: Approximating Area and the Definite Integral
- §5.3: The Fundamental Theorem of Calculus
- §5.4: The Net Change Theorem
- §5.4-5.7: Integration Formulas and the Method of Substitution

1. Compute the integrals below.

(a) $\int_{-1}^{0} (t^{1/3} - t^{2/3})dt = \frac{3}{4}t^{4/3} - \frac{3}{5}t^{5/3}\Big]_{-1}^{0} = (0) - \left(\frac{3}{4} - \left(\frac{3}{5}\right)(-1)\right)$

$= -\left(\frac{3}{4} + \frac{3}{5}\right) = \frac{-(15+12)}{20} = \frac{-27}{20}$

(b) $\int_{0}^{2} x\sqrt{4 - x^2}\, dx = -\frac{1}{2}\int_{4}^{0} u^{1/2} du = -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big]_{4}^{0} = -\frac{1}{3}\left(0^{3/2} - 4^{3/2}\right) = \frac{8}{3}$

$u = 4 - x^2$ | $x = 0,\ u = 4$
$du = -2x\, dx$ | $x = 2,\ u = 0$
$-\frac{1}{2}du = x\, dx$

(c) $\int (x^{2.35} + \frac{3}{4x} + e^x)\, dx = \frac{1}{3.35}x^{3.35} + \frac{3}{4}\ln|x| + e^x + C$

(d) $\int \frac{1}{1 + 4x^2}\, dx = \frac{1}{2}\arctan(2x) + C$

(e) $\int \sec^2(5x) + e^{3x}\, dx = \frac{1}{5}\tan(5x) + \frac{1}{3}e^{3x} + C$

2. Find and simplify the derivative of the function $h(x) = \int_1^{e^t} t^7 \ln(t)\, dt$

$$h'(x) = (e^x)^7 (\ln(e^x)) \cdot e^x = x e^{8x}$$

3. A population of chickadees is changing at a rate of $r(t)$ chickadees per year.

(a) What does $\int_1^4 r(t)\, dt = 400$ mean? Make sure to include units in your answer.

The net change in chickadee population between years 1 and 4 is 400 chickadees.
[The population had a net gain of 400 chickadees between years 1 and 4.

(b) Is it possible for $\int_0^{t_0} r(t)\, dt < 0$ for some time $t_0 > 0$? Explain your answer.

Yes. This would indicate that the population of chickadees was smaller at year x_o that at year zero.

(c) Evaluate $\int_1^4 (5r(t) + 10)\, dt$ $= 5\int_1^4 r(t)\,dt + \int_1^4 10\,dt$

$= 5\,(400) + (4-1)(10)$

$= 2000 + 30 = 2030$

A quick review of main ideas/strategies.

- (§4.7) Optimization
- (§4.3 & 4.5) Derivatives, the Shape of a Graph, and Extrema
- (§4.6 & 4.8) Limits, Asymptotes, and L'Hopital's Rule
- (§4.1) Related Rate Problems
- (§4.10) Initial Value Problems
- (§4.2) Linear Approximations and Differentials

4. A particle is moving with acceleration $a(t) = t + e^{t/2}$ in meters per second per second. You measure that at time $t = 0$, its position is given by $s(0) = 0$ meters and its velocity is given by $v(0) = 8$ meters per second. Determine the position of the particle at time $t = 1$. Determine the average rate of change of the particle between $t = 1$ and $t = 3$.

$a = t + e^{t/2}$

$v = \int a(t)\,dt$

$= \int (t + e^{t/2})\,dt$

$= \frac{1}{2}t^2 + 2e^{t/2} + C$

at $t=0$

$8 = v(0) = 2 + C$

$C = 6$

$v(t) = \frac{1}{2}t^2 + 2e^{t/2} + 6$

$s(t) = \int v(t)\,dt = \int \left(\frac{1}{2}t^2 + 2e^{t/2} + 6\right) dt$

$= \frac{1}{6}t^3 + 4e^{t/2} + 6t + C$

$0 = s(0) = 0 + 4 + 0 + C$. So $C = -4$

$s(t) = \frac{1}{6}t^3 + 4e^{t/2} + 6t - 4$

position at $t=1$: $s(1) = \frac{1}{6} + 4e^{1/2} + 6 - 4$

$= 4e^{1/2} + \frac{13}{6}$ meters

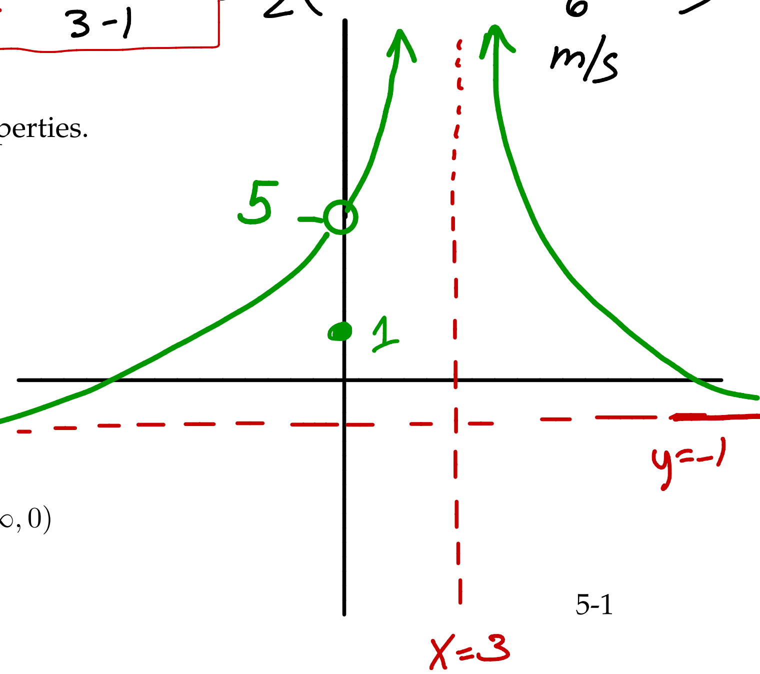
avg vel from $t=1$ to $t=3$ is

!!! $\boxed{\dfrac{s(3)-s(1)}{3-1}} = \frac{1}{2}\left(18.5 + 4e^{3/2} - \frac{13}{6} - 4e^{1/2}\right)$ m/s

5. Sketch a graph $H(x)$ with all of the following properties.

- The domain of $H(x)$ is $(-\infty, 3) \cup (3, \infty)$
- $H(0) = 1$ (0,1) point
- $\lim_{x\to 0} H(x) = 5$ $y \to 5$ when $x \to 0$
- $\lim_{x\to\infty} H(x) = -1$ h.a @ $y=-1$
- $\lim_{x\to 3} H(x) = \infty$ V.A. @ $x=3$
- $H'(x) > 0$ and $H''(x) > 0$ on the interval $(-\infty, 0)$

↑ ccup



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6. The height of a right circular cylinder is increasing at a rate of 2 meters per second while its volume remains constant. At what rate is the radius changing when the radius is 10 meters and height is 20 meters. (Note, the volume of a cylinder is given by $V = \pi r^2 h$ where r is the radius and h is the height of the cylinder.)

$\frac{dh}{dt} = 2$ m/s, $\frac{dV}{dt} = 0$

Find $\frac{dr}{dt}$ when $r = 10, h = 20$

$$V = \pi r^2 h$$

$$0 = \frac{dV}{dt} = \pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$$

$$= \pi\left(2 \cdot 10 \cdot \frac{dr}{dt} \cdot 20 + 10^2 \cdot 2\right)$$

So $0 = 400\frac{dr}{dt} + 200$ or $\frac{dr}{dt} = \frac{-1}{2}$ m/s

7. Find the derivative for each function below.

(a) $F(x) = \dfrac{x - \sin^2(x)}{x + \sin(2x)}$

$$F'(x) = \frac{(x + \sin(2x))(1 - 2\sin(x)\cos(x)) - (x - \sin^2(x))(1 + 2\cos(2x))}{(x + \sin(2x))^2}$$

(b) $g(t) = t\arcsin(t^2)$

$$g'(t) = 1 \cdot \arcsin(t^2) + t \cdot \left(\frac{2t}{\sqrt{1 - t^4}}\right)$$

(c) Find dy/dx for $x\ln(y) = 5 + x^2y^2$.

$$1 \cdot \ln(y) + x \cdot \frac{1}{y}\frac{dy}{dx} = 2xy^2 + 2x^2y\frac{dy}{dx}$$

$$\left(\frac{x}{y} - 2x^2y\right)\frac{dy}{dx} = 2xy^2 - \ln(y); \text{ So } \frac{dy}{dx} = \frac{2xy^2 - \ln(y)}{\frac{x}{y} - 2x^2y}$$

8. Find the following limits

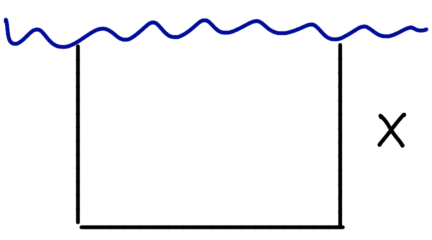
(a) $\lim_{x\to 5} \frac{\frac{1}{x}-\frac{1}{5}}{x-5} = \lim_{x\to 5} \frac{\frac{x-5}{5x}}{x-5} = \lim_{x\to 5} \frac{1}{5x} = 25$

(b) $\lim_{x\to\infty} \frac{2x^3-3x}{4x^2+5x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{2-\frac{3}{x^2}}{\frac{4}{x}+5} = \frac{2}{5}$

(c) $\lim_{x\to 0} \frac{x^2}{1-\cos(x)} \overset{H}{=} \lim_{x\to 0} \frac{2x}{\sin(x)} \overset{H}{=} \lim_{x\to 0} \frac{2}{\cos(x)} = 2$

form $\frac{0}{0}$ form $\frac{0}{0}$

9. A farmer has 400 meters of fencing and wants to fence off a rectangular field that borders a straight river. No fencing is needed along the river, which forms one side of the rectangle. What are the dimensions of the field that has the largest area?



$400 = 2x + y$

$A = xy$

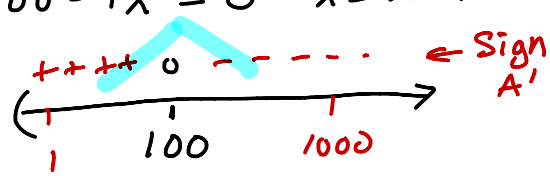
$A = x(400-2x)$

$= 400x - 2x^2$

D: $(0, \infty)$

$A'(x) = 400 - 4x = 0 \quad x = 100\text{ m}$

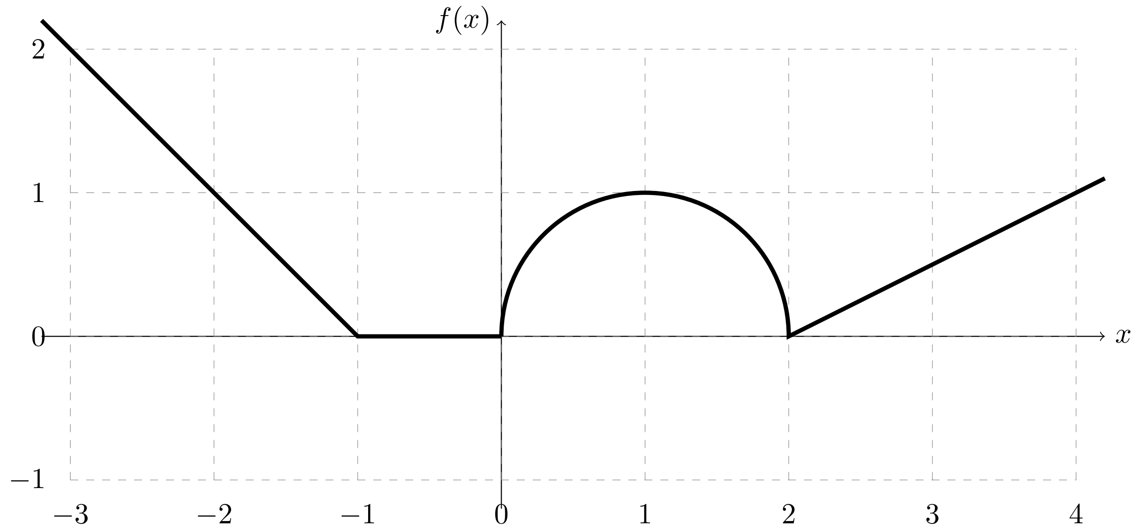
max? Yes.



Answer

$x = 100\text{ m}, \quad y = 200\text{ m}$

10. Consider the function $f(x)$ graphed below. Between $x = 0$ and 2, the graph is of a semicircle of radius 1.



(a) At what x values, if any, does $f'(x)$ not exist?

$x = -1, 0, 2$

(b) What is the value of $f'(-2)$? -1 (slope)

(c) Evaluate $\int_{-1}^{4} f(x)\,dx$. $= 0 + \frac{1}{2}\pi(1)^2 + \frac{1}{2}(2)(1) = \frac{1}{2}\pi + 1$

(d) Let $g(x) = \int_{1}^{x} f(s)\,ds$. What is the value of $g(0)$? $g(0) = \int_1^0 f(s)\,ds = -\int_0^1 f\,ds$

$= -\left(\frac{1}{4}\pi(1)^2\right) = -\frac{\pi}{4}$

(e) For $g(x)$ from part **d.**, what is the value of $g'(4)$.

$g'(4) = f(4) = 1$