REVIEW FOR FINAL EXAM

Topics from Chapter 5:

- §5.1 & 5.2: Approximating Area and the Definite Integral
- §5.3: The Fundamental Theorem of Calculus
- §5.4: The Net Change Theorem
- §5.4-5.7: Integration Formulas and the Method of Substitution

1. Compute the integrals below.

   (a) $\int_{-1}^{0} (t^{1/3} - t^{2/3}) dt$

   (b) $\int_{0}^{2} x\sqrt{4 - x^2}\, dx$

   (c) $\int (x^{2.35} + \frac{3}{4x} + e^x)\, dx$

   (d) $\int \frac{1}{1 + 4x^2}\, dx$

   (e) $\int \sec^2(5x) + e^{3x}\, dx$

2. Find and simplify the derivative of the function $h(x) = \int_1^{e^t} t^7 \ln(t)\, dt$

3. A population of chickadees is changing at a rate of $r(t)$ chickadees per year.

   (a) What does $\int_1^4 r(t)\, dt = 400$ mean? Make sure to include units in your answer.

   (b) Is it possible for $\int_0^{t_0} r(t)\, dt < 0$ for some time $t_0 > 0$? Explain your answer.

   (c) Evaluate $\int_1^4 (5r(t) + 10)\, dt$

A quick review of main ideas/strategies.

- (§4.7) Optimization
- (§4.3 & 4.5) Derivatives, the Shape of a Graph, and Extrema
- (§4.6 & 4.8) Limits, Asymptotes, and L'Hopital's Rule
- (§4.1) Related Rate Problems
- (§4.10) Initial Value Problems
- (§4.2) Linear Approximations and Differentials

4. A particle is moving with acceleration $a(t) = t + e^{t/2}$ in meters per second per second. You measure that at time $t = 0$, its position is given by $s(0) = 0$ meters and its velocity is given by $v(0) = 8$ meters per second. Determine the position of the particle at time $t = 1$. Determine the average rate of change of the particle between $t = 1$ and $t = 3$.

5. Sketch a graph $H(x)$ with all of the following properties.

- The domain of $H(x)$ is $(-\infty, 3) \cup (3, \infty)$
- $H(0) = 1$
- $\lim_{x \to 0} H(x) = 5$
- $\lim_{x \to \infty} H(x) = -1$
- $\lim_{x \to 3} H(x) = \infty$
- $H'(x) > 0$ and $H''(x) > 0$ on the interval $(-\infty, 0)$

6. The height of a right circular cylinder is increasing at a rate of 2 meters per second while its volume remains constant. At what rate is the radius changing when the radius is 10 meters and height is 20 meters. (Note, the volume of a cylinder is given by $V = \pi r^2 h$ where r is the radius and h is the height of the cylinder.)

7. Find the derivative for each function below.

   (a) $F(x) = \dfrac{x - \sin^2(x)}{x + \sin(2x)}$

   (b) $g(t) = t \arcsin(t^2)$

   (c) Find dy/dx for $x \ln(y) = 5 + x^2 y^2$.

8. Find the following limits

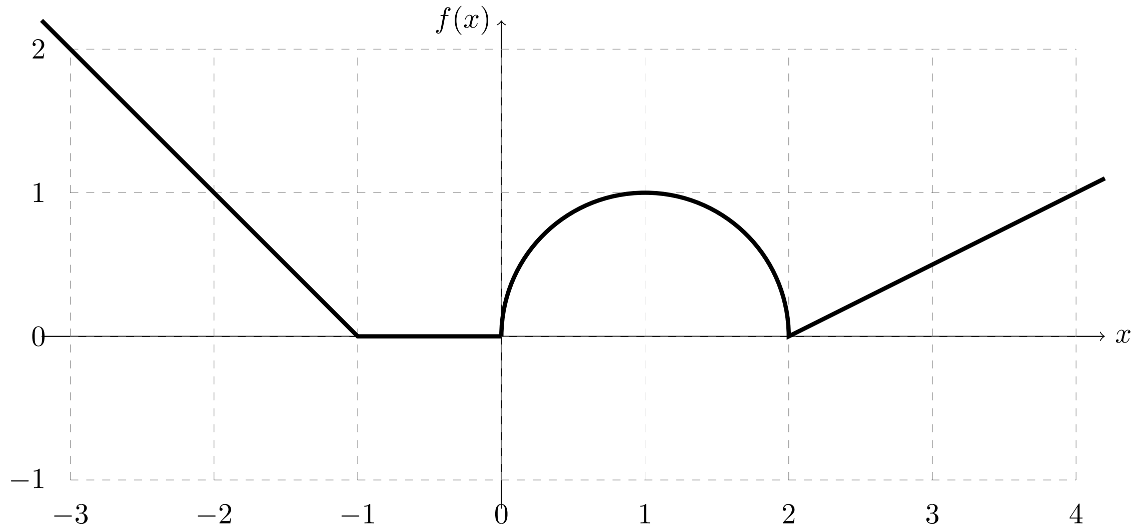
(a) $\lim_{x\to 5} \frac{\frac{1}{x} - \frac{1}{5}}{x - 5}$

(b) $\lim_{x\to\infty} \frac{2x^3 - 3x}{4x^2 + 5x^3}$

(c) $\lim_{x\to 0} \frac{x^2}{1 - \cos(x)}$

9. A farmer has 400 meters of fencing and wants to fence off a rectangular field that borders a straight river. No fencing is needed along the river, which forms one side of the rectangle. What are the dimensions of the field that has the largest area?

10. Consider the function $f(x)$ graphed below. Between $x = 0$ and 2, the graph is of a semicircle of radius 1.



(a) At what x values, if any, does $f'(x)$ not exist?

(b) What is the value of $f'(-2)$?

(c) Evaluate $\int_{-1}^{4} f(x)\,dx$.

(d) Let $g(x) = \int_{1}^{x} f(s)\,ds$. What is the value of $g(0)$?

(e) For $g(x)$ from part **d.**, what is the value of $g'(4)$.