Your Name

Your Signature

Problem	Total Points	Score
1	6	
2	8	
3	6	
4	6	
5	8	
6	10	
7	10	
8	10	
9	10	
10	6	
11	10	
12	10	
extra credit	5	
Total	100	

- You have 2 hours.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids of any kind.
- In order to earn partial credit, you must show your work.
- The last page of the exam contains formulas.

1. (6 points) Let $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{i} - \mathbf{k}$.

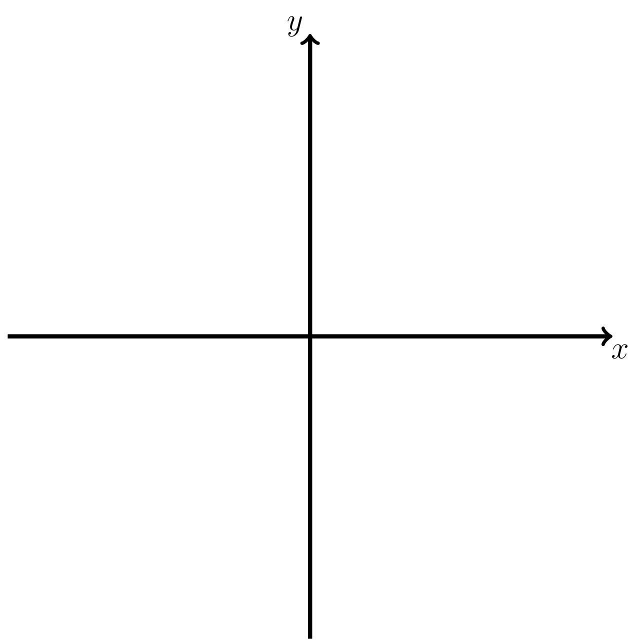
   (a) Find a unit vector that is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

   (b) Find the vector projection of $\mathbf{b}$ onto $\mathbf{a}$: $\text{proj}_{\mathbf{a}}\mathbf{b}$.

2. (8 points) Find an equation of the plane through the point $(1, 2, -1)$ and parallel to the plane $4x - y - 3z = 6$. Simplify to the form $ax + by + cz + d = 0$.

3. (6 points) Find the parametric equations for the tangent line to the curve $\mathbf{r}(t) = t^2\,\mathbf{i} + \sin t\,\mathbf{j} + \cos t\,\mathbf{k}$ when $t = \pi/2$.

4. (6 points) For the surface $f(x, y) = \sqrt{4x^2 + y^2}$, sketch and label the level curves for $z = 0$, $z = 1$, and $z = 2$ on the axes below. Add to your graph, a vector $\mathbf{v}$ such that the derivative of $f(x, y)$ at the point $(0, 1)$ in the direction of $\mathbf{v}$ is zero.



5. (8 points) Find the linearization, $L(x, y)$, of the function $f(x, y) = 5y\sqrt{x}$ at the point $(1, 4)$.

6. (10 points) Let E be the solid bounded by the surfaces $y^2 + z^2 = 9$, $x = -1$, and $x = 1$. Set up, but do not evaluate, the triple integral $\iiint\limits_E f(x, y, z)dV$ using the given order of integration:

   (a) $dx\,dy\,dz$

   (b) $dz\,dy\,dx$

7. (10 points)

   (a) Find all critical points of the function: $f(x, y) = 2 - x^4 + 2x^2 - y^2$.

   (b) Find all local maxima, local minima, and saddle points of the function in part **(a)**.

8. (10 points) Find the mass of the lamina that occupies the region D in the first quadrant bounded by $y = 0$, $y = \sqrt{3}\,x$, and $x^2 + y^2 = 9$ and has density function $\rho(x, y) = (x^2 + y^2)^{3/2}$.

9. (10 points) Find the volume of the solid which is above the cone $z = \sqrt{x^2 + y^2}$ and inside (below) the sphere $x^2 + y^2 + z^2 = 1$.

10. (6 points) Match the vector fields **F** with the plots labeled I,II,III,IV:

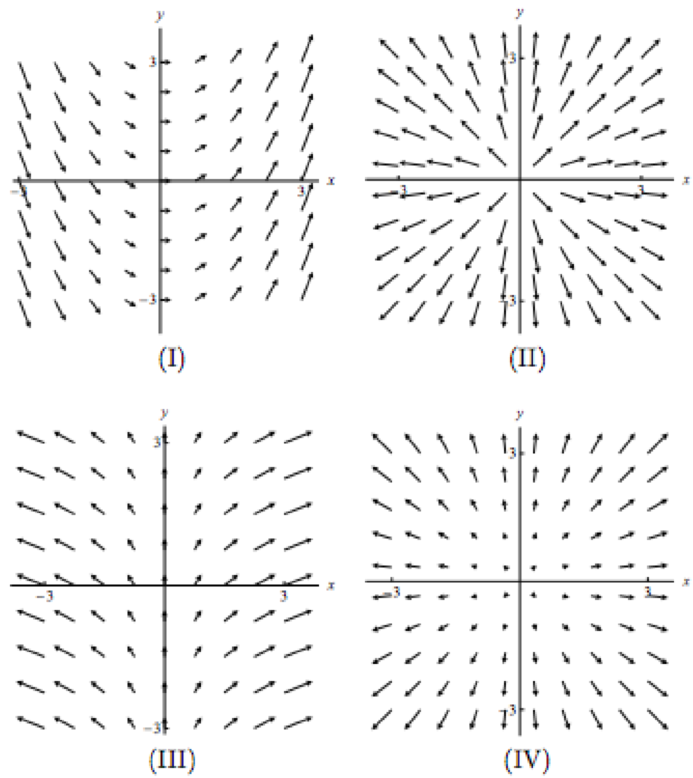
(a) $\mathbf{F}(x, y) = \langle x, 1 \rangle$ ______ ⟵ *write "I", "II",... in the spaces*

(b) $\mathbf{F}(x, y) = \langle 1, x \rangle$ ______

(c) $\mathbf{F}(x, y) = \nabla f$ ______

where $f(x, y) = x^2 + y^2$

(d) $\mathbf{F}(x, y) = \left\langle \frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}} \right\rangle$ ______



11. (10 points) Find the work done by the force field $\mathbf{F}(x,y) = (1+xy)e^{xy}\,\mathbf{i} + x^2e^{xy}\,\mathbf{j}$ in moving a particle along the line segment from $(-1,2)$ to $(3,0)$.

12. (10 points) Suppose $\mathbf{F}(x, y) = \langle x^2 + y, 3x - y^2 \rangle$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for *any* positively-oriented, closed, simple curve C enclosing a region D which has area 10. (*Hint.* Green's Theorem)

Extra Credit (5 points) Assume $\mathbf{F}(x, y)$ is a conservative vector field defined on a open, connected region D. Fix any point (a, b) in D. Explain why the formula

$$f(x, y) = \int_{(a,b)}^{(x,y)} \mathbf{F} \cdot d\mathbf{r}$$

defines a function f on D. Explain why this function satisfies $\nabla f = \mathbf{F}$.