Your Name

Your Signature

Problem	Total Points	Score
1	15	
2	5	
3	8	
4	8	
5	10	
6	10	
7	15	
8	8	
9	6	
10	15	
extra credit	5	
Total	100	

- You have 1 hour to complete the midterm.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids of any kind.
- In order to earn partial credit, you must show your work.

1. (15 points) Given vectors $\mathbf{v} = 2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{j} + 2\mathbf{k}$, answer the questions below.

   (a) Find a unit vector parallel to $\mathbf{w}$.

   (b) Find a vector $\mathbf{u}$ orthogonal to both $\mathbf{v}$ and $\mathbf{w}$.

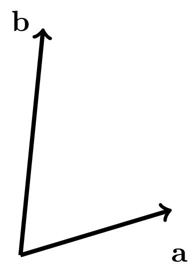
   (c) Find $\text{proj}_{\mathbf{v}}\mathbf{w}$.

   (d) Determine if the angle between $\mathbf{v}$ and $\mathbf{w}$ is acute, right, or obtuse. Show that your answer is correct.

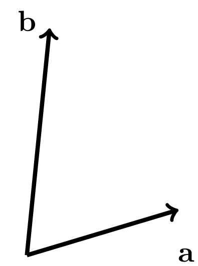
2. (5 points) Describe in words or draw the region of $\mathbb{R}^3$ represented by the inequality $x^2 + y^2 \leq 2$.

3. (8 points)Use the pictures of the vectors **a** and **b** below to draw the following vectors.

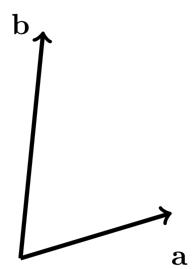
(a) $\frac{-1}{2}\mathbf{a}$



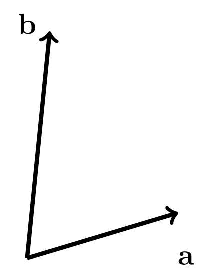
(b) $\mathbf{b}+\mathbf{a}$



(c) $\mathbf{b}-\mathbf{a}$



(d) $\text{proj}_{\mathbf{a}}\mathbf{b}$

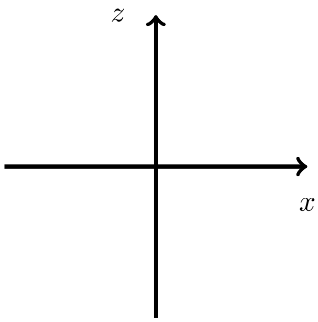


4. (8 points) Write the equation of the sphere that passes through the point (2, 4, -1) and has center (1, 2, -3).

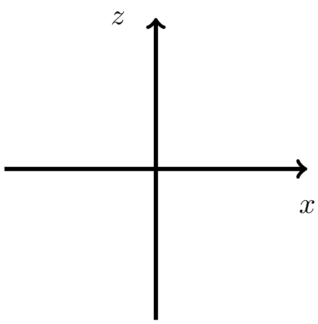
5. (10 points) Find equations for the line through $(-2, 2, 4)$ perpendicular to the plane $2x+5z = 12+y$.

6. (10 points) Find an equation of a plane through $(1, 2, -2)$ that contains the line $x = 2t,\ y = 3 - t,\ z = 1 + 3t$.

7. (15 points) Use traces to sketch and identify the surface $2x^2 + z^2 = y^2 - 2$ **Label your curves.**
(a) The traces for $y = 0$, $y = 2$, and $y = 4$



(b) The traces for $z = 0$, $z = 1$ and $z = 2$.



(c) Identify the surface. You may use a sketch, a verbal description including its proper name. I recommend all three. (Note that you may choose to make additional traces, if you like.)

8. (10 points) Find the unit tangent vector $\mathbf{T}(t)$ at time $t = 0$ for the vector-valued function $\mathbf{r}(t) = \langle \sin(2t), e^{3t}, te^{t} \rangle$.

9. (8 points) Find the length of the curve $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ between $t = 0$ and $t = 1$. (Set up the integral only. You do not need to evaluate it.)

10. (15 points) Assume a projectile is fired from a position 100 meters above the ground with an initial speed of 500 meters per second and angle of elevation 30 degrees. Find vector-valued functions for the acceleration, velocity and position of the projectile in terms of time t. Assume $t = 0$ when the projection is fired. Note that acceleration due to gravity is $9.8\,m/s^2$.

Extra Credit (5 points): Find a vector-valued function that represents the curve of intersection of $x^2 + y^2 = 16$ and $x + z = 5$.