Your Name

Your Signature

Problem	Total Points	Score
1	10	
2	15	
3	10	
4	15	
5	10	
6	15	
7	10	
8	15	
extra credit	5	
Total	100	

- You have 1 hour to complete the midterm.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids of any kind.
- In order to earn partial credit, you must show your work.
- Formulas:
  - $A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$
  - $z = \rho\cos\phi$, $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $dV = \rho^2\sin\phi\, d\rho\, d\theta\, d\phi$.

1. (10 points) Evaluate the triple integral:

$$\int_0^{\pi} \int_0^{1} \int_0^{\sqrt{1-y^2}} y \sin(x)\, dz\, dy\, dx$$

2. (15 points) Evaluate the double integral below. [HINT: Change the order of integration.]

$$\int_0^4 \int_{\sqrt{x}}^2 \sqrt{y^3+1}\, dy\, dx$$

3. (10 points) Change the integral below into cylindrical coordinates:

$$\int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{16-x^2-y^2} \sqrt{x^2+y^2}\, dz\, dy\, dx$$

4. (15 points) Let E be the solid that lies above the xy-plane and below the paraboloid $z = 2 - x^2 - y^2$. Assume E has density function $\rho(x, y, z) = x^2 + z^4$.

   (a) Set up but do not evaluate the integral expressions for the mass, m, of the solid.

   (b) Set up but do not evaluate the expression for $\overline{z}$, the z-coordinate of the center of mass of E.

5. (10 points) Convert the point $(x, y, z) = (2, 2, -2)$ in rectangular coordinates to spherical coordinates.

6. (15 points) Rewrite the integral below in *spherical* coordinates. The expressions in your answer must be simplified.

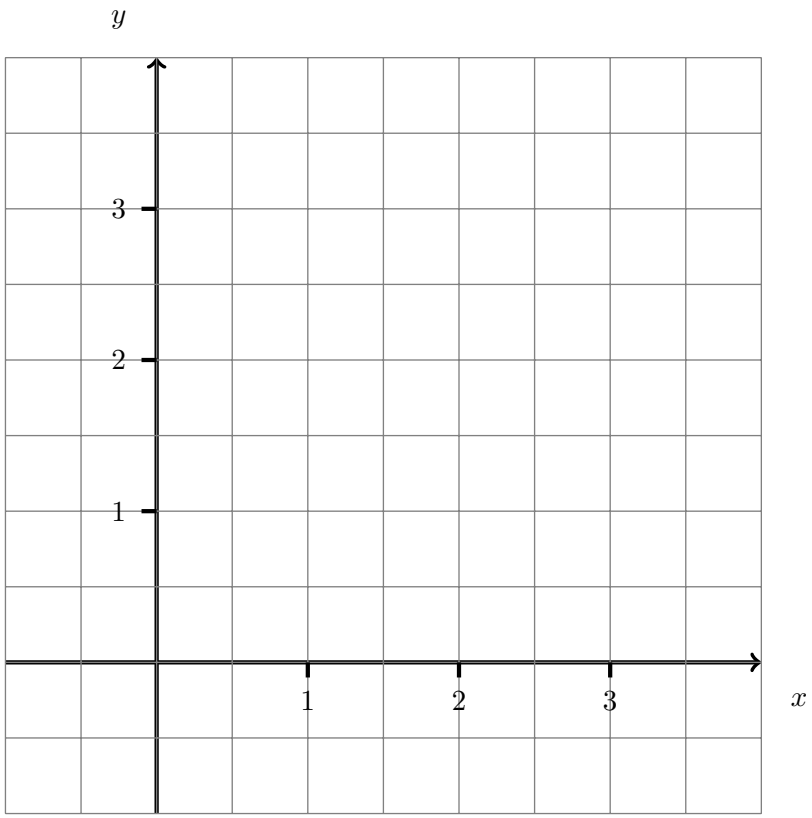
$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2+y^2+z^2}\, dz\, dx\, dy$$

7. (10 points) On the graph below, make a rough sketch of the vector field

$$\mathbf{F}(x, y) = (x + y)\mathbf{i} + (x - y)\mathbf{j}$$

Your sketch does not have to be perfect or to scale, but your vectors should be in roughly the correct direction and *relative* length. Make sure to include vectors at:

- at least three points along the positive x-axis
- at least three points along the postive y-axis
- the points $(1, 1)$, $(2, 2)$, $(3, 3)$



8. (15 points) Let E be the solid bounded below by the paraboloid $z = x^2 + y^2$ and above by the half-cone $z = \sqrt{x^2 + y^2}$. Set up but do not evaluate a triple integral to find the volume of this solid. Pick a coordinate system for which this triple integral is as simple as possible to evaluate.

Extra Credit: (5 points) Rewrite the triple integral

$$\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} f(x, y, z)\, dz\, dy\, dx$$

in the order $dx\, dy\, dz$.