Your Name

Solutions

Your Signature

Problem	Total Points	Score
1	6	
2	6	
3	6	
4	18	
5	12	
6	20	
7	16	
8	10	
9	6	
extra credit	5	
Total	100	

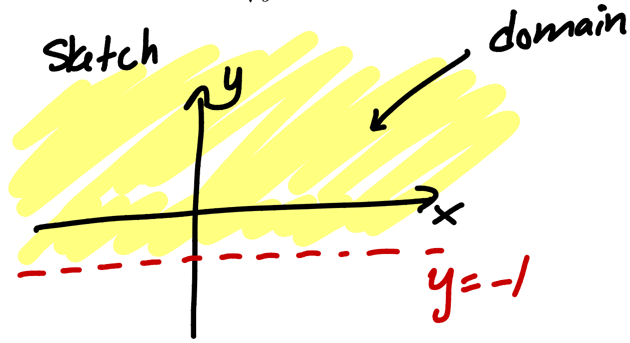
- You have 1 hour to complete the midterm.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids of any kind.
- In order to earn partial credit, you must show your work.

1. (6 points) Find and sketch the domain of the function $f(x, y) = \frac{\sin x}{\sqrt{y+1}}$.

Need $y+1>0$

or $y>-1$

domain: $\{(x,y) \mid y>-1\}$

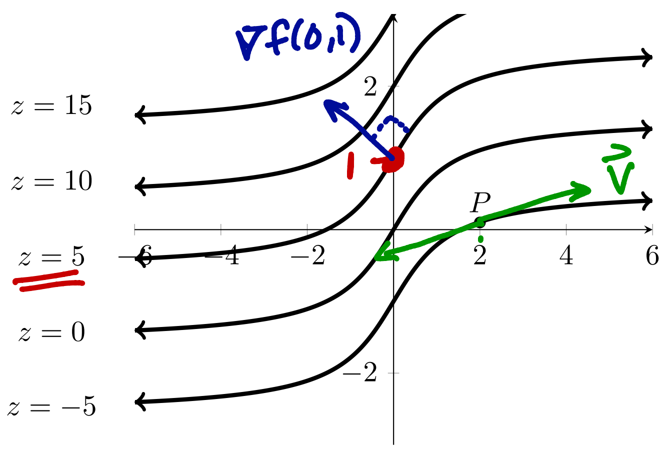


2. (6 points) A model for the surface area of the human body, S in square feet, is given as a function of weight, w in pounds, and height, h in inches. (That is $S = f(w, h)$.) Interpret in the context of the problem the meaning of

$$S_h(160, 70) = 0.2.$$

$S_h(160,70)$ tells us that if a 70 in-tall person weighing 160 pounds grows by 1 inch in height - assuming his/her weight is constant - that person can expect his/her surface area to increase by about 0.2 square feet

3. (6 points) A contour map of the function $z = f(x, y)$ is graphed below. Use it to answer the following questions.



(a) Find $f(0, 1)$. (answer: 5) (Red dot is at $x=0$, $y=1$)

(b) On the contour diagram sketch a vector in the direction of $\nabla f(0, 1)$. $\nabla f(0,1) \perp$ level curve

(c) On the contour diagram sketch a vector $\mathbf{v}$ at the point P such that the directional derivative of f at P in the direction of $\mathbf{v}$ would be zero.

$\vec{v}$ should be tangent to level curve.

Either arrow is acceptable

4. (18 points) Let $f(x, y) = x^2 e^{3y}$.

(a) Find the linearization, $L(x, y)$, of f at the point $(1, 0)$.

$f(0,1) = 1.$

$f_x = 2x e^{3y}$

$f_x(0,1) = 2$

$f_y = 3x^2 e^{3y}$

$f_y(0,1) = 3$

$z - z_0 = f_x \cdot (x - x_0) + f_y \cdot (y - y_0)$

$z - 1 = 2(x-1) + 3(y-0)$

$z = L(x,y) = 1 + 2(x-1) + 3y$

(b) Find $L(1.1, 0.2)$.

$L(1.1, 0.2) = 1 + 2(0.1) + 3(0.2) = 1.8$

(c) Explain in geometric or numerical terms (or both) what the linearization $L(x, y)$ at a point (x_0, y_0).

some possible answers. One is sufficient.

- $L(x,y)$ is the plane tangent to the surface $f(x,y)$ at point (x_0,y_0).
- $L(x,y)$ is an approximation of $f(x,y)$ near the point of tangency, (x_0, y_0).
- $L(x,y) \approx f(x,y)$ if (x,y) is close to (x_0, y_0)

5. (12 points)

(a) If $T = f(u, v)$, $u = g(p, q, r)$ and $v = h(p, q, r)$, write the Chain Rule to find $\partial T/\partial p$.

$$\frac{\partial T}{\partial p} = \frac{\partial T}{\partial u} \cdot \frac{\partial u}{\partial p} + \frac{\partial T}{\partial v} \cdot \frac{\partial v}{\partial p}$$

(b) If $T = \frac{u}{v}$, $u = pq^2r$, and $v = pq + r$, find $\partial T/\partial p$ when $p = 1$, $q = -1$ and $r = 2$.

$$\frac{\partial T}{\partial p} = \left(\frac{1}{v}\right)(q^2 r) + \left(-1 \cdot u \cdot v^{-2}\right)(q)$$

$u = 1 \cdot (-1)^2 \cdot 2 = 2$

$v = 1 \cdot (-1) + 2 = 1$

at $(p,q,r) = (1,-1,2)$, $\frac{\partial T}{\partial p} = \left(\frac{1}{1}\right)(1 \cdot 2) - \left(\frac{2}{1}\right)(-1) = 2 + 2 = 4$

6. (20 points) Let $f(x, y, z) = 3xy + \cos z$.

(a) Find the gradient of $f(x, y, z)$.

$\nabla f(x,y,z) = \langle 3y, 3x, -\sin z \rangle$

(b) Find the directional derivative of the function f at the point $(2, 3, 0)$ in the direction of vector $\mathbf{v} = \langle 1, 4, -1 \rangle$.

$\nabla f(2,3,0) = \langle 3\cdot 3, 3\cdot 2, -\sin 0 \rangle = \langle 9,6,0 \rangle$; $|\vec{v}| = \sqrt{18} = 3\sqrt{2}$

So, $\vec{u} = \left\langle \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}, \frac{-1}{3\sqrt{2}} \right\rangle$

$D_{\vec{u}} f(2,3,0) = \langle 9,6,0 \rangle \cdot \left\langle \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}, \frac{-1}{3\sqrt{2}} \right\rangle = \frac{9+24}{3\sqrt{2}} = \frac{33}{3\sqrt{2}} = \frac{11}{\sqrt{2}}$

(c) Find the maximum rate of change of f at the point $(2, 3, 0)$.

$|\nabla f(2,3,0)| = |\langle 9,6,0 \rangle| = \sqrt{81+36} = \sqrt{117}$

(d) Find an equation of the tangent plane of the level surface $3xy + \cos z = \cancel{13}\ 19$ at the point $(2, 3, 0)$.

$\nabla f(2,3,0) = \langle 9,6,0 \rangle$ (from a)

plane: $9(x-2) + 6(y-3) + 0(z-0) = 0$

or

$9(x-2) + 6(y-3) = 0$ or

$9x + 6y = 36$

7. (16 points)Let $f(x,y) = x^3 + y^3 - 3x^2 - 3y^2 - 9x$.

(a) Find all critical points of f.

$f_x(x,y) = 3x^2 - 6x - 9$	$f_{xx}(x,y) = 6x - 6$	$f_{xy}(x,y) = 0$
$f_y(x,y) = 3y^2 - 6y$	$f_{yy}(x,y) = 6y - 6$	$f_{yx}(x,y) = 0$

Set $f_x = 0$ and $f_y = 0$.

$f_x = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x-3)(x+1) = 0$. So $\underline{x=-1}$ or $\underline{x=3}$

$f_y = 3y^2 - 6y = 3y(y-2) = 0$. So $\underline{y=0}$ or $\underline{y=2}$

crit. pts: $(-1,0), (-1,2), (3,0), (3,2)$

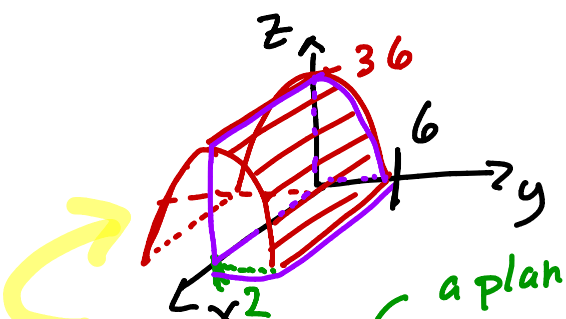
(b) Identify *the locations of* the local minimum and maximum values and saddle point(s) of the function. Note that you do not need to actually find the maximum or minimum values.

points	$D = (6x-6)(6y-6)$	$f_{xx} = (6x-6)$	conclusion
$(-1,0)$	$(-)(-) = + > 0$	$-$ (a negative or "<0") $\cap$ ccdown	local max at $(-1,0)$
$(-1,2)$	$(-)(+) < 0$	$\sim$	saddle at $(-1,2)$
$(3,0)$	$(+)(-) < 0$	$\sim$	saddle @ $(3,0)$
$(3,2)$	$(+)(+) > 0$	$+$ $\cup$ ccup	local min at $(3,2)$

8. (10 points) Evaluate $\int_0^3 \left(\int_0^{\pi/2} (y + y^2 \cos x)\, dx\right) dy = \int_0^3 \left[yx + y^2 \sin x \right]_{x=0}^{x=\pi/2} dy$

$$= \int_0^3 \left(\frac{\pi}{2} y + y^2\right) dy = \frac{\pi}{4} y^2 + \frac{1}{3} y^3 \Big]_{y=0}^{y=3} = \frac{9\pi}{4} + 9$$

using $\sin \frac{\pi}{2} = 1$ and $\sin 0 = 0$



a plane slicing of "front" of tunnel

only parts where $x, y, z \geq 0$

9. (6 points) Find the volume of the solid in the first octant bounded by the cylinder $z = 36 - y^2$ and the plane $x = 2$. **Set up the double integral only. You do not need to evaluate it.**

$$V = \int_0^2 \int_0^6 (36 - y^2)\, dy\, dx \overset{\text{or}}{=} \int_0^6 \int_0^2 (36 - y^2)\, dx\, dy$$

[Extra Credit] (5 points) **Use Lagrange Multipliers** to find the maximum and the minimum of the function $f(x, y) = xy^2$ subject to the constraint $2x + 6y = 8$.

$\nabla f(x,y) = \lambda \nabla g(x,y)$ gives (i) $y^2 = \lambda 2$ and (ii) $2xy = \lambda 6$.

So $\lambda = \frac{y^2}{2} = \frac{2xy}{6}$. So either: (a) $y = 0$ or (b) $y = \frac{4x}{6} = \frac{2x}{3}$

For (a): If $y = 0$, then $x = 4$. Point $\underline{\underline{(4,0)}}$.

For (b): If $y = 2x/3$, then $2x + 6(2x/3) = 8$ or $6x = 8$ or $x = 4/3$

And $y = 8/9$. point $(4/3, 8/9)$

Answer: $f(4,0) = 0$ ← minimum

$f(4/3, 8/9) = \frac{4}{3}\left(\frac{64}{81}\right)$ ← maximum