# Review for Final Exam

Preliminaries:
The final exam is on Monday December 10 from 10:15am-12:15pm in our usual Monday classroom. It is closed-book and closed-note. Calculators are not allowed.
The best preparation is to *work* problems – since this is what your test will look like. Where to find problems: midterms, quizzes, homework from this course, old midterms and finals.

material appearing on Midterm 1

## Chapter 12: Vectors and the Geometry of Space

- Section 1
  1. terminology: coordinate planes
  2. important ideas: basic graphing in 3D, distance formula in 3D, equation of a sphere, regions of $\mathbb{R}^3$ described by simple algebraic expressions (example: the set of points satisfying $x = 2$ OR the set of point satisfying $x = 2$ and $y = 1$ OR the set of points $z \geq -4$.
- Section 2
  1. terminology: vector versus scalar, components of a vector, standard basis vectors
  2. important ideas: algebraic and geometric interpretation of vector addition, vector subtraction and scalar multiplication, parallel vectors
  3. no "sum of force" applied problems except possibly as extra credit
- Section 3
  1. definitions & theorems: dot product of two vectors, Theorem 3 connecting the dot product with the angle between two vectors, orthogonal vectors, scalar projection and vector projection
  2. skills: compute the dot product and know its geometric interpretation, know how to use the dot product to find the angle between vectors or to determine if two vectors are orthogonal, know how to compute the scalar/vector projection of one vector onto another and their geometric interpretations,
  3. no work problems or orthogonal projection problems except possibly as extra credit
- Section 4
  1. definitions & theorems: cross product, scalar triple product, Theorem 9 relating the cross product and the angel between vectors,
  2. skills: know how to compute the cross product (scalar triple product) and know its geometric interpretations related to (i) orthogonality, (2) angle between vectors, (3) parallel vectors and (4) area or volumes.
  3. no torque problems except possibly as extra credit

- Section 5

1. terminology: normal to a plane, parametric(vector, symmetric) forms of a line
2. skills: know how to write the equations of lines and planes, know how to use equations of lines and planes regardless of the form in which they are given. (example: Find the equation of the plane containing three points P, Q and R. OR Find the equations of the line containing points P and Q.), know how to find points of intersection
3. You do not need to memorize the formula for the distance between a point and a plane. However, GIVEN a particular point and a particular plane you should be able calculate this distance. (example: Given the point P(1,1,1) and the plane $-x + 2y - z = 1$, find the distance between the two.)

- Section 6

1. terminology: cylinders and quadric surfaces, traces, planes parallel to coordinate planes, ellipsiod, cone, elliptic(hyperbolic) paraboloid, hyperboloid (of one or two sheets).
2. skills: know how to find and draw specific traces of a surface, know how to put traces together to identify/describe surfaces.

CHAPTER 13:

- Section 1

1. terminology: vector-valued function, parameter
2. skills: know how to use and sketch the graph of a (simple) vector-valued function,

- Section 2

1. skills: know how to differentiate and integrate vector-valued functions, know how to interpret the derivatives and integrals of vector-valued functions

- Section 3

1. definitions: arc length, arc length function, unit tangent vector
2. skills: know how to find the arc length of a vector-valued function, know how to build the arc length function of a vector-valued function given a starting point, know how to parametrize a function with respect to arc length and how to interpret the result
3. no questions about the unit normal or binormal

- Section 4

1. skills: know how to find velocity and acceleration of a particle in 3D given its position function, know how to find the position of a particle given its acceleration (or velocity) and appropriate initial conditions, know how to model projectile motion in 2D given appropriate information. (I'll give you the acceleration of gravity constant in the right units.)

material appearing on Midterm 2

## CHAPTER 14: PARTIAL DERIVATIVES

- Section 1: Functions of Several Variables

  1. terminology: level curves, level surfaces
  2. important ideas: the nature of domain and range for a function of multiple variables, graphing functions of multiple variables especially using level curves or level surfaces,
  3. In particular, you should know how to sketch a contour diagram (or contour map) and how to read a contour diagram.

- Section 2: Limits and Continuity

  1. terminology: $\lim_{(x,y)\to(a,b)} f(x,y)$,
  2. important ideas: how to demonstrate that a limit fails to exist, how we can know that a limit does exist, how we know certain functions are continuous.

- Section 3: Partial Derivatives

  1. terminology: first- (or second- or third-) order partial derivative, $f_{xxyz}$ or $\partial^2 f/\partial x\partial y$,
  2. important ideas and skills: how to find a first-order partial derivative and how to interpret it, especially in the context of an applied problem, how to demonstrate that a particular function does or does not satisfy a particular differential equation.
  3. One may use Clairaut's Theorem though you do not need to know it by name.

- Section 4: Tangent Planes and Linear Approximation

  1. definitions: tangent plane to a surface at a point, linear approximation of a function at a point, the differential (or total differential) of a function
  2. skills: how to find the tangent plane to a surface at a point (NOTE: We now have multiple ways of doing this.), how to find the linear approximation of a function at a point and how to use it, how to find the differential of a function at a point and how to use it.
  3. In particular, you should know how to find and use these things (plane, linear approximation, differential) in the context of an applied problem.

- Section 5: The Chain Rule

  1. skill: Given a function of multiple variables each of which is itself a function of other variables, you should be able to state an appropriate chain rule and to use it.
  2. You may use the Implicit Function Theorem but are not required to do so.

- Section 6: Directional Derivatives and the Gradient Vector

  1. terminology: directional derivative a given direction, gradient
  2. skills: know how to find and interpret a directional derivative in a given direction, know the geometric significance of the gradient and the magnitude of the gradient at a point (summarized in the *Significance of the Gradient Vector* at the end of this section.
  3. Recall that we have an alternate approach to finding tangent planes to surfaces.

- Section 7: Maximum and Minimum Values

  1. terminology: local (absolute) maximum (minimum) for functions of multiple variables, critical points, Second Derivatives Test, closed bounded set.
  2. skills: how to find critical points for a function of two variables, how to use the Second Derivatives Test, how to find absolute extrema for a continuous function on a closed-bounded domain.

- Section 6: Lagrange Multipliers

  1. terminology: Method of Lagrange Multipliers
  2. skill: how to solve a max-min problem using the Method of Lagrange Multipliers

material appearing on Midterm 3

CHAPTER 15: MULTIPLE INTEGRALS

- Section 1: Double Integrals over Rectangles

  1. terminology: iterated integral, double integral, Riemann sum
  2. skills: how to evaluate a double integral (by straight evaluation, by reversing the order of integration, by geometry), how to reverse the order of integration, how to set up and evaluate a double integral to find volume of a solid.

- Section 2: Double Integrals over General Regions

  1. You should know how to set up and evaluate a double integral over a non-rectangular region.
  2. You should know how to reverse the order of integration.
  3. You should know how to use a double integral to find volume.
  4. No question will use or require the language or "type I" or "type 2" regions.

5. Typical problem types: #1-10 (easy),15-32, 45-56

- Section 3: Double Integrals in Polar Coordinates

  1. You should know how to set up an integral in polar coordinates.
  2. You should know how to convert integrals between polar and rectangular coordinates.
  3. Typical problem types: #5-27, 29-32

- Section 4: Applications of Double Integrals

  1. We looked exclusively at using double integrals to find the mass and center of mass of a lamina with a given density. You should be comfortable with the notation and language of these types of problems including $M_x$ and $M_y$.
  2. Typical problems: #3-16

- Section 5: Surface Integrals

  1. We learned how to find the area of a surface with some given domain.
  2. I will give you the formula: $A(S) = \iint\limits_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$

- Section 6: Triple Integrals

  1. You should know how to set up and evaluate a triple integral.
  2. No question will use or require the language of "type I", "type 2" or "type 3" regions.
  3. You should know how to use a triple integral to find the mass and center of mass of a solid. You should be comfortable with the notation and language of these types of problems including $M_{xy}$, $M_{xz}$ and $M_{yz}$.
  4. typical problems: #3-22, 27-32, 35-26, 39-42
  5. Note that you will *not* be asked to give all six different orders of integration but you should expect to be asked to set up or rewrite a triple integral in some subset of these.

- Sections 7 and 8: Triple Integrals in Cylindrical and Spherical Coordinates

  1. You should be able to plot points and graph surfaces given in cylindrical and spherical coordinates.
  2. You should be able to convert points (equations, integrals) between rectangular and cylindrical and spherical coordinates.
  3. I will give you the formulas given to you on Quiz 9:

     | Formulas for Spherical Coordinates: |
     |---|
     | $z = \rho\cos\phi$, $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $dV = \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$. |

  4. You should be able to set up and evaluate integrals in cylindrical and spherical coordinates.
  5. typical problems: 15.7 #1-12, 15-24, 29-30
  6. typical problems: 15.8 #1-14, 17-27, 41-43

CHAPTER 16: VECTOR CALCULUS

- Section 1: Vector Fields

  1. You should be able to give a rough sketch of a given (not-too-complicated) vector field or identify which among a collection of plots is a given vector field.
  2. You should know that the gradient of a function can be viewed as a vector field.
  3. You should know what a *conservative* vector field is.
  4. typical problems: #1-14, 29-32

  Post Midterm 3

- Section 2: Line Integrals

  1. You should know how to evaluate a line integral over a scalar field in any of its forms:
  $$\int_C f(x,y)\; ds \text{ or } \int_C P(x,y)\; dx + Q(x,y)\; dy \text{ or } \int_C f(x,y,z)\; ds \cdots$$
  2. You should know effective simple way to parametrize common curves.
  3. You should know how to evaluate a line integral over a vector field.
  4. You should know how to covert between line integrals over scalar and vector fields (see page 1124)
  5. You should know how to interpret line integrals geometrically and in terms of work.

- Section 3: The Fundamental Theorem for Line Integrals

  1. You should know and know how to use the Fundamental Theorem of Line Integrals (page 1127) though not by name. This is the theorem that states that – if a vector field is conservative – line integrals my be evaluated by "plugging in" the end points of the curve.
  2. You should know what **path independence** means for line integrals.
  3. You should know how to "show" that a vector field is conservative.

- Section 4: Green's Theorem

  1. You should know and know how to apply Green's Theorem and to do so by name. (See problems 1-4 for example.)
  2. You should know the $\oint$ notation.

- Section 5: Curl and Divergence

  1. You should know how to compute the curl and the divergence of a vector field.
  2. You should be comfortable using $\vec{\nabla}$ notation.
  3. You should know how to use the curl of a vector field to show it is conservative.