Math 253 Calculus III Fall 2018 **Quiz # 1, 5 Sept 2018**

Name: Solution

There are 20 points possible on this quiz. This is a closed book quiz and closed note quiz. Calculators are not allowed. If you have any questions, please raise your hand.

1. (2 points each) Use vectors $\vec{a} = 3\vec{i} - 3\vec{j} + \vec{k}$ and $\vec{b} = -\vec{i} + 6\vec{k}$ answer the questions below.

12.2 #19,22

(a) Find $|\vec{a}|$ $= \sqrt{3^2+(-3)^2+1^2} = \sqrt{9+9+1} = \sqrt{19}$

(b) Find $\vec{a} - 3\vec{b}$

$\vec{a} - 3\vec{b} = \langle 3,-3,1\rangle + \langle 3,0,-18\rangle = \langle 6,-3,-17\rangle$

(c) Find $\vec{a} \cdot \vec{b}$

12.3.5 $\langle 3,-3,1\rangle \cdot \langle -1,0,6\rangle = -3+0+6 = 3$

(d) Find a **unit** vector, $\vec{u}$, in the direction *opposite* vector $\vec{a}$.

12.2.24 $\vec{u} = \frac{-\vec{a}}{|\vec{a}|} = \frac{-1}{\sqrt{19}}\langle 3,-3,1\rangle = \left\langle \frac{-3}{\sqrt{19}}, \frac{3}{\sqrt{19}}, \frac{-1}{\sqrt{19}} \right\rangle$

(e) Find a vector, $\vec{w}$, of length 5 in the direction of vector $\vec{b}$.

12.2.26 $\vec{w} = 5 \cdot \frac{\vec{b}}{|\vec{b}|} = \frac{5}{\sqrt{37}}\langle -1,0,6\rangle = \left\langle \frac{-5}{\sqrt{37}}, 0, \frac{30}{\sqrt{37}} \right\rangle$

(f) Determine if vector $\vec{c} = \langle 2,4,-4\rangle$ is orthogonal to vector $\vec{a}$. You must show your work to receive credit.

12.3.25 $\vec{a} \cdot \vec{c} = \langle 3,-3,1\rangle \cdot \langle 2,4,-4\rangle = 6-12-4 = -10 \neq 0$ So $\vec{a}$ is not orthogonal to $\vec{b}$.

(g) Find the scalar projection of $\vec{b}$ onto $\vec{a}$.

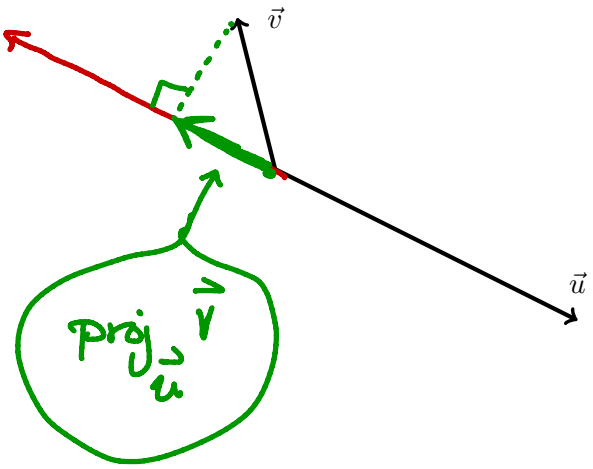
12.3.39 & 44 $\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{3}{\sqrt{19}}$

(h) Find the vector projection of $\vec{b}$ onto $\vec{a}$.

$\text{proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\right)\vec{a} = \frac{3}{19}\langle 3,-3,1\rangle = \left\langle \frac{9}{19}, \frac{-9}{19}, \frac{3}{19} \right\rangle.$

12.3.46

2. (2 points each) Let vectors $\vec{u}$ and $\vec{v}$ be graphed below.



(a) In the drawing above, sketch the vector projection of $\vec{v}$ onto $\vec{u}$. Clearly indicate your answer.

(b) Would the scalar projection of $\vec{v}$ onto $\vec{u}$ be positive, negative or zero? Explain your answer.

negative.

- proj$_{\vec{u}}$ $\vec{v}$ points in the direction opposite $\vec{u}$.

or

- Since $90° < \theta$, $\cos\theta < 0$. Thus $\vec{a} \cdot \vec{b} < 0$.