Math 253 Calculus III Fall 2018 **Quiz # 10, 28 November 2018**

Name: Solutions

There are 20 points possible on this quiz. This is a closed book quiz and closed note quiz. Calculators are not allowed. If you have any questions, please raise your hand.

1. Assume C is the upper half of the unit circle $x^2 + y^2 = 1$.

   (a) (2 points) Give a complete parametrization of C.

   $x = \cos t$, $y = \sin t$, $0 \le t \le \pi$

   (b) (2 points) Assume $\int_C (2 + x^2 y)\, ds = 2\pi + \frac{2}{3}$. Explain what this means geometrically. Be specific.

   The area under the surface $z = 2 + x^2 y$ above the curve C (half circle) is $2\pi + 2/3$ units.

2. (8 points) Evaluate the line integral $\int_C yz\cos x\, ds$ where C is the curve parametrized by $x = t$, $y = 3\cos t$ and $z = 3\sin t$ for $0 \le t \le \frac{\pi}{2}$.

$$ds = \sqrt{1^2 + (-3\sin t)^2 + (3\cos t)^2}\, dt = \sqrt{1+9}\, dt = \sqrt{10}\, dt.$$

$$\text{So } \int_C yz\cos x\, ds = \sqrt{10}\int_0^{\pi/2} (3\cos t)(3\sin t)(\cos t)\, dt$$

$$= 9\sqrt{10}\int_0^{\pi/2} \cos^2 t \sin t\, dt = -3\sqrt{10}\,(\cos t)^3\Big]_0^{\pi/2}$$

$$= -3\sqrt{10}\left[\cos^3(\pi/2) - (\cos 0)^3\right]$$

$$= 3\sqrt{10}$$

3. (a) (6 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \cancel{xy\,\mathbf{i} + x^2\,\mathbf{j}}$ $e^{2x}\vec{i} + xy\vec{j}$ and C is given by $\mathbf{r}(t) = \cancel{\sin t\,\mathbf{i} + (1+t)\,\mathbf{j}}$ $t^3\vec{i} + (1+t)\vec{j}$ for $0 \le t \le 1$.

$$\vec{F}(t) = \langle e^{2t^3}, t^3(1+t) \rangle = \langle e^{2t^3}, t^3 + t^4 \rangle$$

$$\vec{r}\,'(t) = \langle 3t^2, 1 \rangle$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 \left(3t^2 e^{2t^3} + t^3 + t^4\right) dt = \frac{1}{2} e^{2t^3} + \frac{1}{4} t^4 + \frac{1}{5} t^5 \Big]_0^1$$

$$= \frac{1}{2} e^2 + \frac{1}{4} + \frac{1}{5} - \frac{1}{2} = \frac{10e^2 - 1}{20}$$

(b) (2 points) Interpret your answer from part (a).

$\int_C \vec{F} \cdot d\vec{r}$ calculates the work done by force field $\vec{F}$ on the particle moving along C.