Math 253 Calculus III Fall 2018 **Quiz # 11, 5 December 2018**

Name: Solutions

There are 20 points possible on this quiz. This is a closed book quiz and closed note quiz. Calculators are not allowed. If you have any questions, please raise your hand.

1. (7 points) Let $\mathbf{F}(x, y, z) = 2x \sin y\, \mathbf{i} + (x^2 \cos y + e^z)\, \mathbf{j} + (ye^z + 1)\, \mathbf{k}$.

   (a) Find a potential for $\mathbf{F}$.

$$f(x,y,z) = x^2 \sin y + ye^z + z$$

check: $f_x = 2x\sin y$, $f_y = x^2\cos y + e^z$, $f_z = ye^z + 1$ ✓

   (b) Use part (a) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve: $x = t + 1,\ y = \pi t$, and $z = t^2$ for $0 \le t \le 1$.

at $t=0$: $x=1, y=0, z=0$ $(1,0,0)$

at $t=1$: $x=2, y=\pi, z=1$ $(2,\pi,1)$

Since $\vec{F}$ is conservative, the FT of Line Integrals applies:

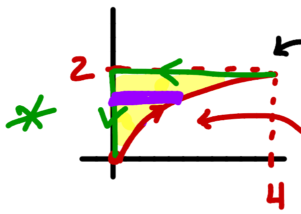
$$\int_C \vec{F} \cdot d\vec{r} = f(2,\pi,1) - f(1,0,0)$$

$$= (2^2 \cdot \sin\pi + \pi e^1 + 1) - (1^2 \sin 0 + 0e^0 + 0)$$

$$= \pi e + 1$$

*See next page for alternate order of integration →

2. (7 points) Evaluate $\oint_C xy\ \ dx + x^2y\ \ dy$ where C consists of the curve $y = \sqrt{x}$ from $(0,0)$ to $(4,2)$ followed by the line segments from $(4,2)$ to $(0,2)$ and from $(0,2)$ to $(0,0)$.



← C is positively oriented.

$y=\sqrt{x}$ or $x=y^2$

$$\oint_C xy\,dx + x^2y\,dy = \iint_D (2xy - x)\,dA$$

$$\int_0^2\int_0^{y^2}(2xy-x)\,\underline{\underline{dx\,dy}} = \int_0^2 \left. x^2y - \tfrac{1}{2}x^2\right]_{x=0}^{x=y^2} dy = \int_0^2 y^5 - \tfrac{1}{2}y^4\,dy$$

$*$

$$= \left.\tfrac{1}{6}y^6 - \tfrac{1}{10}y^5\right]_{y=0}^{y=2} = \tfrac{1}{6}(2)^6 - \tfrac{1}{10}(2)^5 - (0) = \frac{2^5}{3} - \frac{2^4}{5} = 16\left(\tfrac{2}{3}-\tfrac{1}{5}\right) = 7.4\overline{6}$$

3. (6 points) Let $\mathbf{F} = ye^x\ \mathbf{i} + e^x\ \mathbf{j} + xz\ \mathbf{k}$.

(a) Find curl $\mathbf{F}$.

$$\text{curl}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ ye^x & e^x & xz \end{vmatrix} = (0)\vec{i} - (z-0)\vec{j} + (e^x - e^x)\vec{k}$$

$$= -z\vec{j}$$

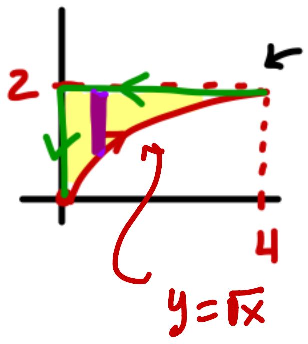
(b) Find div $\mathbf{F}$.

$$\text{div}\,\vec{F} = \frac{\partial}{\partial x}(ye^x) + \frac{\partial}{\partial y}(e^x) + \frac{\partial}{\partial z}(xz)$$

$$= ye^x + 0 + x = \underline{\underline{x + ye^x}}$$

~~(c) Is F conservative? Explain.~~

2. (7 points) Evaluate $\oint_C xy \quad dx + x^2y \quad dy$ where C consists of the curve $y = \sqrt{x}$ from $(0,0)$ to $(4,2)$ followed by the line segments from $(4,2)$ to $(0,2)$ and from $(0,2)$ to $(0,0)$.



← C is positively oriented.

$$\oint_C xy\,dx + x^2y\,dy = \iint_D 2xy - x \; dA$$

$$= \int_0^4 \int_{\sqrt{x}}^2 x(2y-1)\,dy\,dx = \int_0^4 x\,(y^2-y)\Big]_{\sqrt{x}}^2 dx = \int_0^4 x\big((4-2)-(x-\sqrt{x})\big)dx$$

$$= \int_0^4 x\left(2-x+x^{1/2}\right)dx = \int_0^4 2x - x^2 + x^{3/2}\,dx = x^2 - \frac{1}{3}x^3 + \frac{2}{5}x^{5/2}\Big]_{x=0}^{x=4}$$

$$= 4^2 - \frac{1}{3}\cdot 4^3 + \frac{2}{5}(4)^{5/2} - (0)$$

$$= 16 - \frac{64}{3} + \frac{64}{5} = 7.4\overline{6}$$

3. (6 points) Let $\mathbf{F} = ye^x\,\mathbf{i} + e^x\,\mathbf{j} + xz\,\mathbf{k}$