Math 253 Calculus III Fall 2018 **Quiz # 2, 12 Sept 2018**

Name: Solutions

There are 20 points possible on this quiz. This is a closed book quiz and closed note quiz. Calculators are not allowed. If you have any questions, please raise your hand.

1. (6 points) Given points $P(0,-2,0)$, $Q(4,1,\ \ 2)$, and $R(5,3,1)$ in $\mathbb{R}^3$. Answer the questions below.

   (a) Find a nonzero vector orthogonal to the plane through points P, Q, and R.

$\vec{PQ} = \langle 4-0, 1-(-2), 2-0\rangle = \langle 4, 3, 2\rangle$

$\vec{PR} = \langle 5-0, 3-(-2), 1-0\rangle = \langle 5, 5, 1\rangle$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & 2 \\ 5 & 5 & 1 \end{vmatrix} = (3-10)\vec{i} - (4-10)\vec{j} + (20-15)\vec{k}$$

$$= -7\vec{i} + 6\vec{j} + 5\vec{k}$$

   (b) Find the area of triangle PQR.

$$\text{area } \Delta PQR = \frac{1}{2}|\vec{n}| = \frac{\sqrt{49+36+25}}{2} = \frac{\sqrt{110}}{2}$$

2. (6 points) Find equations (parametric, vector, or symmetric) for the line through the point $P(-2,5,8)$ and parallel to line L_2 with parametric equations: $x = 3-2t,\ y = 4t,\ z = 9$.

L_2 has direction $\vec{d} = \langle -2, 4, 0\rangle$

using point $P(-2,5,8)$:

<u>parametric</u>

$x = -2-2t$

$y = 5-4t$

$z = 8$

<u>vector</u>

$\vec{r} = \langle -2, 5, 8\rangle + t\langle -2, 4, 0\rangle$

3. (6 points) Find an equation of the plane that contains the line $\vec{r}(t) = \langle -1, 1, 0\rangle + t\langle 1, 4, -2\rangle$ and is parallel to the plane $z = 3 - 6x + y$.

plane: $6x - y + z = 0$. So $\vec{n} = \langle 6, -1, 1\rangle$

line $\vec{r}$ contains the point $P(-1, 1, 0)$

answer: $6(x-(-1)) + (-1)(y-1) + 1(z-0) = 0$

So $6(x+1) - (y-1) + z = 0$ or

$6x - y + z = -7$

4. (2 points) State whether each expression is meaningful. If not, explain why. If so, state whether it is a vector or a scalar.

(a) $(\vec{a} \times \vec{b}) \times \vec{c}$

meaningful, vector

(a) $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$ not meaningful.

$\vec{a} \cdot \vec{b}$ is a scalar, but the dot product requires vectors as inputs.

(b) $(\vec{a} \cdot \vec{b}) \times (\vec{c} \cdot \vec{d})$ not meaningful

$\vec{a} \cdot \vec{b}$ and $\vec{c} \cdot \vec{d}$ give scalars.
But the cross product requires vectors as inputs.

(b) $(a \times b) \cdot (c \times d)$
meaningful, scalar