Math 253 Calculus III Fall 2018 Quiz # 3, 19 Sept 2018

Name: Solutions

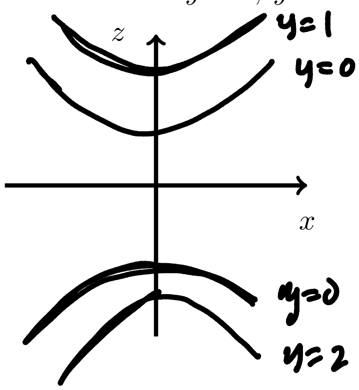
There are 20 points possible on this quiz. This is a closed book quiz and closed note quiz. Calculators are not allowed. If you have any questions, please raise your hand.

1. (2 points each) For the surface $z^2 - x^2 - 4y^2 = 4$, sketch the traces below *if the traces exist.* Label your graphs. Note axes have been given and labelled for you.

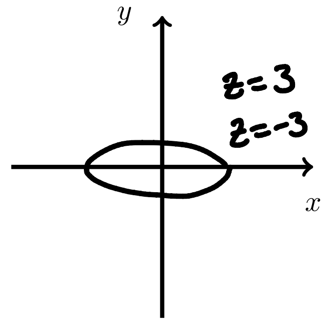
   (a) The traces for $y = 0$, $y = 1$

   $y=0$: $z^2-x^2=4$

   $y=1$: $z^2-x^2=8$



   (b) The traces for $z = -3$, $z = 0$, and $z = 3$.



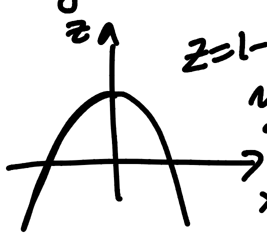
   $z=-3$: $5 = x^2+4y^2$

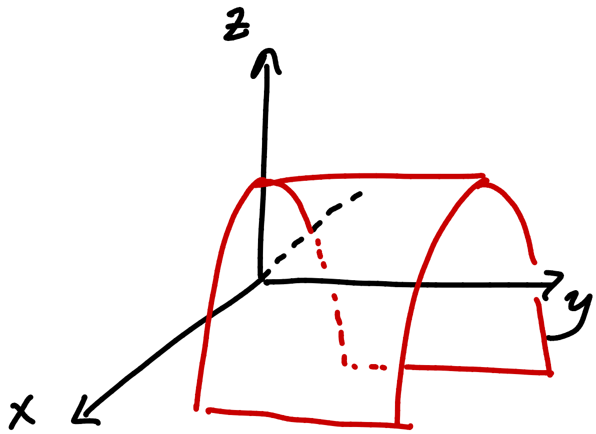
   $z=0$: $-x^2-4y^2=4$ no soln

2. (2 points) Describe the surface $z = 1 - x^2$. Your description can be in words or with a rough sketch. I recommend both.

cylinder

$z=1-x^2$
y - anything





In words, up-side-down parabolic-shaped trough openning down the negative z-axis. The trough is parallel to y-axis

3. (4 points) Find any points where the curve $\vec{r}(t) = t\,\vec{i} + (2t - t^2)\vec{k}$ intersects the paraboloid $z = x^2 + y^2$.

$\vec{r}$: $\left.\begin{array}{l} x = t \\ y = 0 \\ z = 2t - t^2 \end{array}\right)$ curve.

plug into paraboloid:

$$2t - t^2 = t^2 + 0^2$$

So $0 = 2t^2 - 2t = 2t(t-1)$

So $t = 0$ or $t = 1$.

answer:

$\vec{r}$ intersects paraboloid when

$t = 0$ at point $(0,0,0)$

and

$t = 1$ at point $(1,0,1)$

4. (5 points) For the curve $\vec{r}(t) = \langle \sqrt{t^2+3}, t, \ln(t^2+1) \rangle$, find parametric equations for the tangent line to the curve at the point $(2, 1, \ln(2))$.

when $t = 1$, $\vec{r}(1) = \langle 2, 1, \ln 2 \rangle$. ← point

$$\vec{r}\,'(t) = \left\langle \tfrac{1}{2}(t^2+3)^{-1/2}(2t),\ 1,\ \frac{2t}{t^2+1} \right\rangle = \left\langle \frac{t}{\sqrt{t^2+3}},\ 1,\ \frac{2t}{t^2+1} \right\rangle$$

$$\vec{r}\,'(1) = \left\langle \frac{1}{2}, 1, \frac{2}{2} \right\rangle = \left\langle \tfrac{1}{2}, 1, 1 \right\rangle$$ ← direction vector

answer: $x = 2 + \frac{1}{2}t$

$y = 1 + t$

$z = \ln 2 + t$

5. (4 points) Evaluate the integral $\int_0^4 (2t^{3/2}\,\vec{i} + \vec{j} + e^{2t}\vec{k})dt$

$$= \left. \frac{2 \cdot 2}{5} t^{5/2}\,\vec{i} + t\,\vec{j} + \frac{1}{2}e^{2t}\vec{k} \right]_0^4$$

$$= \left\langle \frac{4}{5}4^{5/2}, 4, \frac{1}{2}e^8 \right\rangle - \left\langle 0, 0, \frac{1}{2} \right\rangle$$

$$= \left\langle \frac{128}{5}, 4, \frac{1}{2}e^8 - \frac{1}{2} \right\rangle$$