

Name: Solutions

There are 20 points possible on this quiz. This is a closed book quiz and closed note quiz. Calculators are not allowed. If you have any questions, please raise your hand.

1. (5 points) Find all second order partial derivatives of  $f(x, y) = xe^{2y} - x^2y^{-1}$ .

$$f(x, y) = xe^{2y} - x^2y^{-1}$$

$$f_x(x, y) = e^{2y} - 2xy^{-1}$$

$$f_{xx}(x, y) = -2y^{-1}$$

$$f_{xy}(x, y) = 2e^{2y} + 2xy^{-2}$$

$$f_y(x, y) = 2xe^{2y} + x^2y^{-2}$$

$$f_{yy}(x, y) = 4xe^{2y} - 2x^2y^{-3}$$

$$f_{yx}(x, y) = 2e^{2y} + 2xy^{-2}$$

2. (2 points) Find  $\partial P / \partial V$  for the ideal gas law:  $PV = mRT$ .

$$P = mRT V^{-1}$$

$$\frac{\partial P}{\partial V} = -mRT V^{-2}$$

$$f(x,y) = (1+2y)(2+x)^{-1}$$

3. (5 points) Find the tangent plane to the surface  $f(x,y) = \frac{1+2y}{2+x}$  at the point  $(-3, 4)$ .

$$f(-3,4) = \frac{1+2 \cdot 4}{2-3} = \frac{9}{-1} = -9 = z$$

Using:

$$z - z_0 = (f_x)(x - x_0) + (f_y)(y - y_0)$$

$$f_y(x,y) = \frac{2}{2+x}, \quad f_y(-3,4) = -2$$

$$z + 9 = -9(x + 3) + (-2)(y - 4)$$

$$f_x(x,y) = -1(1+2y)(2+x)^{-2}$$

or

$$f_x(-3,4) = -1(1+2 \cdot 4)(2-3)^{-2}$$

$$9x + 2y + z + 28 = 0$$

$$= -9$$

4. Let  $f(x,y) = x^3 - x \cos(y) + y$ .

(a) Find the differential of  $f(x,y)$ .

$$df = \underbrace{(3x^2 - \cos y)}_{f_x} dx + \underbrace{(x \sin y + 1)}_{f_y} dy$$

(b) Use the differential estimate the change in  $f(x,y)$  as  $(x,y)$  changes from  $(1, 0)$  to  $(0.9, 0.2)$ .

$$\left. \begin{array}{l} dx = 0.9 - 1 = \underline{\underline{-0.1}} \quad dy = 0.2 - 0 = \underline{\underline{0.2}} \\ \underline{\underline{x=1}}, \underline{\underline{y=0}} \end{array} \right] \begin{array}{l} \text{plug these} \\ \text{into} \\ \text{differential} \end{array}$$

$$\begin{aligned} \Delta f \approx df &= (3 \cdot 1^2 - \cos 0)(-0.1) + (1 \cdot \sin 0 + 1)0.2 \\ &= (2)(-0.1) + (1)(0.2) \\ &= -0.2 + 0.2 = \underline{\underline{0}} \end{aligned}$$