Math 253 Calculus III Fall 2018 **Quiz # 6, 17 October 2018**

Name: ______________________________

There are 20 points possible on this quiz. This is a closed book quiz and closed note quiz. Calculators are not allowed. If you have any questions, please raise your hand.

1. (4 points) Use the Chain Rule to find $\partial z/\partial t$ if $z = y^2 \arctan(2x)$, $x = e^{st}$, $y = t^2 + s^3$.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} = \left(\frac{2y^2}{1+4x^2}\right)\left(se^{st}\right) + \left(2y\arctan(2x)\right)(2t)$$

$$= \frac{2y^2 se^{st}}{1+4x^2} + 4yt\arctan(2x)$$

2. (6 points) The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = 3\cos(2\pi t)$, $y = 4 + \sqrt{t}$ where x and y are measured in centimeters. The temperature function satisfies $T_x(3, 5) = 8$ and $T_y(3, 5) = -6$.

   (a) In the context of the problem (temperature, crawling bug), explain the meaning of $T_x(3, 5) = 8$ in language your parents could understand.

   If the bug is at position (3,5) and moves in the positive x direction, it can expect the temperature to rise at a rate of 8°C per cm.

   (b) How fast is the temperature changing on the bug's path after 1 second? (Give units with your answer.)

$$\frac{dT}{dt} = \frac{\partial T}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial T}{\partial y}\cdot\frac{dy}{dt} = \frac{\partial T}{\partial x}\cdot\left(-6\pi\sin(2\pi t)\right) + \frac{\partial T}{\partial y}\cdot\frac{1}{2}t^{-1/2}$$

at $t=1$, $x=3$, $y=5$

$$\left.\frac{dT}{dt}\right|_{t=1} = 8(0) + (-6)\left(\tfrac{1}{2}\right) = -3\ {}^\circ\mathrm{C}/\mathrm{sec}$$

3. (4 points) Find the equation of the tangent plane to the surface $x = y^2 + z^2 + 1$ at the point

point $\rightarrow$ $(14, 2, 3)$.

$0 = -x + y^2 + z^2 + 1 = f(x,y,z)$

$\nabla f(x,y,z) = \langle -1, 2y, 2z \rangle$

$\nabla f(14,2,3) = \langle -1, 4, 6 \rangle$

$= \vec{n}$

plane:

$\boxed{-1(x-14) + 4(y-2) + 6(z-3) = 0}$

or $12 = -x + 4y + 6z$

4. (6 points) Suppose that over a certain region of space the electircal potential V is given by the following equation:

$$V(x, y, z) = \quad xy^2 + \; yz.$$

(a) Find the rate of change of the potential at the point $P(-1, 2, 4)$ in the direction of the vector $\mathbf{v} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

$|\vec{v}| = 3$. So, $\vec{u} = \langle \frac{2}{3}, \frac{-2}{3}, \frac{1}{3} \rangle$

$\nabla V = \langle y^2, 2xy + z, y \rangle$

$\nabla V(-1,2,4) = \langle 4, -4+4, 2 \rangle$

$= \langle 4, 0, 2 \rangle$

$D_{\vec{u}} V(-1,2,4) = \nabla V \cdot \vec{u}$

$= 4 \cdot \frac{2}{3} + 0 \cdot \frac{-2}{3} + 2 \cdot \frac{1}{3} = \frac{8}{3} + \frac{2}{3} = \boxed{\frac{10}{3}}$

answer

(b) In which direction does V change most rapidly at P?

$\langle 4, 0, 2 \rangle = \nabla V(-1,2,4)$

(c) What is the maximum rate of change of V at P?

$|\nabla V| = \sqrt{4^2 + 0^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$